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ETH Lecture 401-0674-00L Numerical Methods for Partial Differential Equations

Course Video
Section 10.1.1: Modeling Fluid Flow

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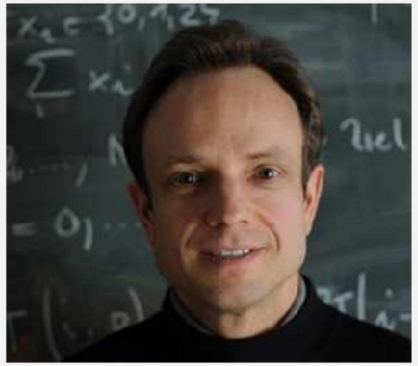


Date: April 26, 2022

Prerequisites.

- Basic knowledge about ordinary differential equations

Dependencies. [Lecture → Section 6.1]

Duration: minutes

Video and accompanying tablet notes may not match completely!

[Corrections and updates may have been made in tablet notes.]

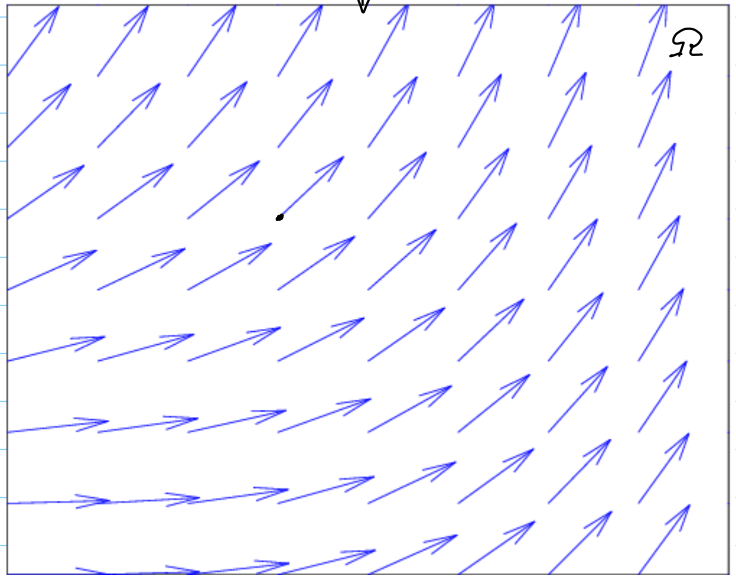
X. Convection-Diffusion Problems

10.1. Heat Conduction in a Fluid

10.1.1. Modeling Fluid Flow (stationary)

$\Omega \subset \mathbb{R}^d \triangleq$ bounded computational domain, $d = 1,2,3$

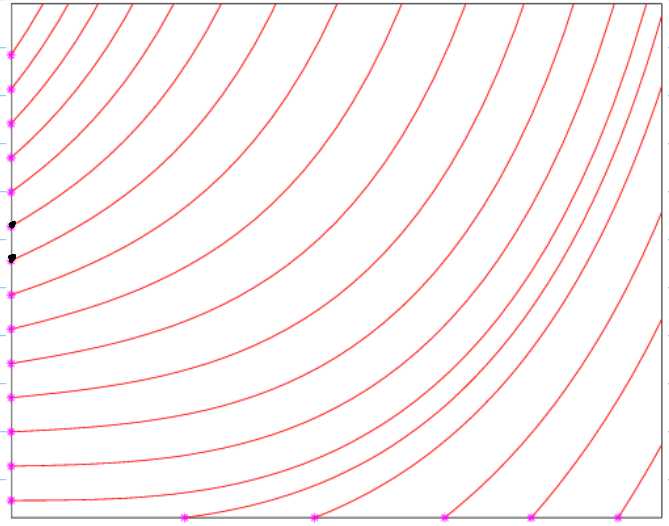
Movement of fluid inside Ω can be described by a flow field $\mathbf{v}: \Omega \to \mathbb{R}^d \triangleq$ velocity field



Streamline ODE

$$\dot{\mathbf{y}} = \mathbf{v}(\mathbf{y}) \qquad (*)$$

Particle trajectories (streamlines) ▷

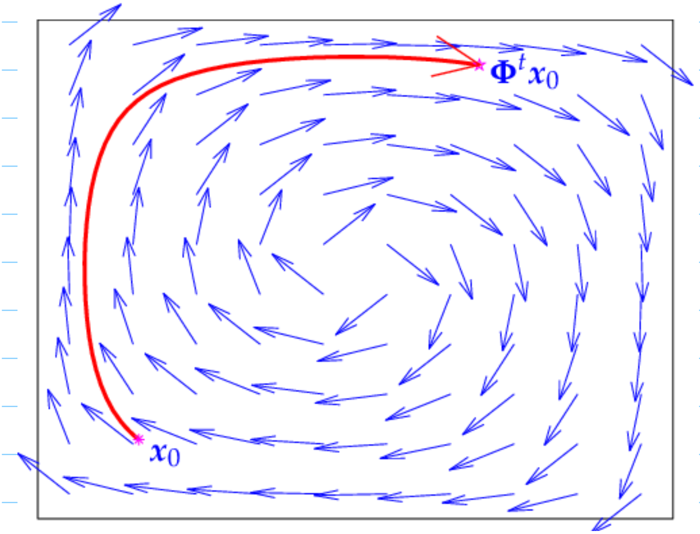


② § 10.1.1.3 (Flow map) $\longleftrightarrow$ evolution op. for $(*)$

Assume $\mathbf{v}\cdot\mathbf{n} = 0$ on $\partial\Omega$ $\Rightarrow$ no in/outflow

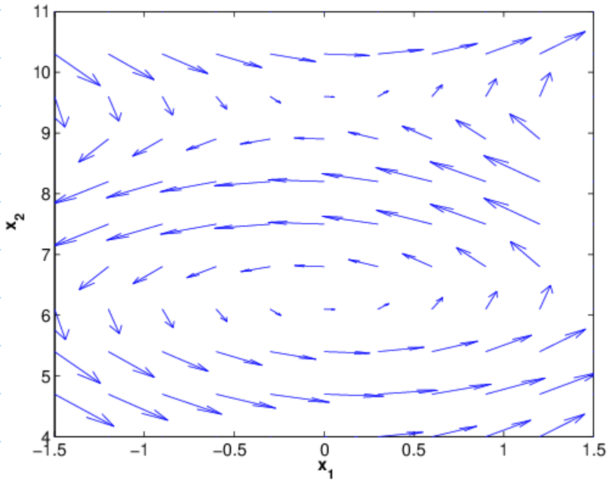
$\Rightarrow$ Streamlines exist for all times

Flow map $\quad \Phi : \begin{cases} \mathbb{R}\times\Omega \longrightarrow \Omega \\ (t, \mathbf{y}_0) \longrightarrow \Phi^t \mathbf{y}_0 := \mathbf{y}(t) \end{cases}$ where $t \mapsto \mathbf{y}(t)$ solves IVP for $(*)$ $\mathbf{y}(0) = \mathbf{y}_0$

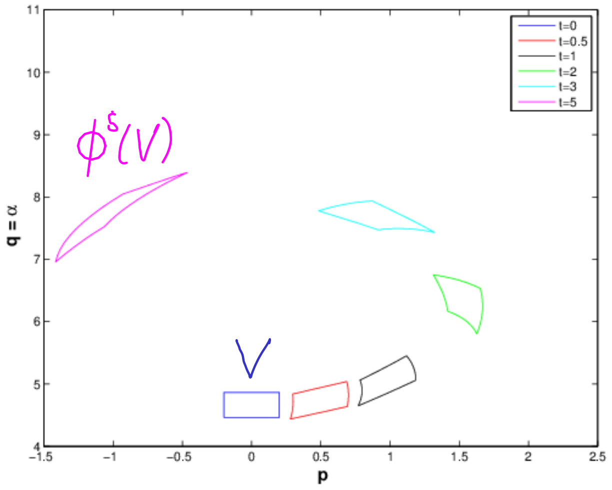


$$\Phi^0 \mathbf{y}_0 = \mathbf{y}_0 \quad \forall \mathbf{y}_0 \in \Omega$$

$$\Phi^{s+t} = \Phi^s \circ \Phi^t$$



flow field $\mathbf{v} : \Omega \mapsto \mathbb{R}^2$



snapshots of $\Phi^t(V)$ for control volume V