ETH Lecture 401-0674-00L Numerical Methods for Partial Differential Equations

Course Video
Section 10.1.2: Heat Convection and Diffusion

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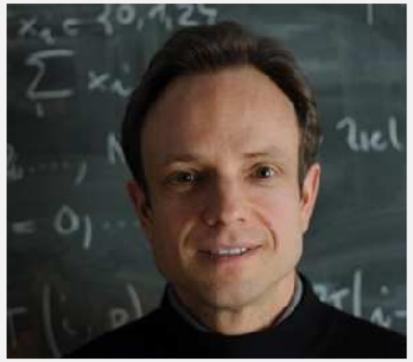

Dependencies. [Lecture → Section 1.6], [Lecture → Section 1.7]

Duration: minutes

Note: Possible minor *mismatch of video and tablet notes*!

[Corrections and updates can be incorporated into tablet notes only]

X. Convection-Diffusion Problems

10.1. Heat Conduction in a Fluid

10.1.2. Heat Convection and Diffusion

given : "Container" $\Omega \subset \mathbb{R}^d$, $d = 1, 2, 3$

Stationary fluid velocity field $\underline{v} : \Omega \to \mathbb{R}^d$

Sought : BVP governing temperature field $u : \Omega \to \mathbb{R}$

Same reasoning as for heat conduction in a solid :

Conservation of energy

$$\operatorname{div} \mathbf{j} = f \iff \int_{\partial V} \mathbf{j} \cdot \mathbf{n} \, dS = \int_V f \, d\mathbf{x} \quad \text{for all "control volumes" } V. \qquad (1.6.0.3)$$

power flux through surface of V

heat production inside V

New : Heat flux due to transport of stored thermal energy

Simple model : $= \rho u$

$\rho > 0$: heat capacity

Fourier's law in moving fluid

$$\mathbf{j}(\boldsymbol{x}) = -\kappa\,\mathbf{grad}\,u(\boldsymbol{x}) + \mathbf{v}(\mathbf{x})\rho u(\boldsymbol{x}), \quad \boldsymbol{x}\in\Omega. \tag{10.1.2.4}$$

diffusive heat flux
(due to spatial variation of temperature)

convective heat flux
(due to fluid flow)

§ 10.1.2.7 (Convection-diffusion equation for temperature)

$$\operatorname{div}\mathbf{j} = f \quad + \quad \mathbf{j}(\boldsymbol{x}) = -\kappa\,\mathbf{grad}\,u(\boldsymbol{x}) + \mathbf{v}(\mathbf{x})\rho u(\boldsymbol{x})$$

$$-\operatorname{div}(\kappa\,\mathbf{grad}\,u) + \operatorname{div}(\rho\mathbf{v}(\boldsymbol{x})u) = f \quad \text{in} \quad \Omega\,. \tag{10.1.2.8}$$

≙ *Linear* scalar **convection-diffusion equation** (CDE)

$$\underbrace{-\operatorname{div}(\kappa\,\mathbf{grad}\,u)}_{\substack{\text{diffusive term}\\ \text{(2nd-order)}}} + \underbrace{\operatorname{div}(\rho\mathbf{v}(\boldsymbol{x})u)}_{\substack{\text{convective term}\\ \text{(1st-order)}}} = f\,.$$

§ 10.1.2.9 (Boundary conditions)

"Nothing new": ↳ same as for std. 2nd-order scalar elliptic PDEs

Suitable boundary conditions fitting a PDE are determined by the highest-order term.

↓ Here: diffusion term

▷ Review questions 10.1.2.10 !