ETH Lecture 401-0674-00L Numerical Methods for Partial Differential Equations

Course Video

Section 10.1.4: Time-Dependent (Transient) Heat Flow in a Fluid

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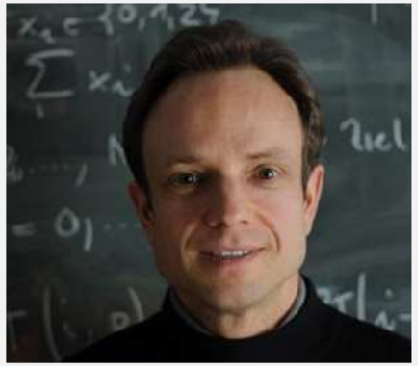

Dependencies. [Lecture → Section 9.2.1]

Duration: minutes

Note: Possible minor *mismatch of video and tablet notes*!

[Corrections and updates can be incorporated into tablet notes only]

X. Convection-Diffusion Problems

10.1. Heat Conduction in a Fluid

10.1.4. Transient Heat Flow in a Fluid

$\Omega \subset \mathbb{R}^d \;\hat{=}\;$ bounded computational domain ("container")

Given: Time-dependent velocity field $\underline{v} = \underline{v}(\underline{x}, t)$, $\underline{x} \in \Omega$

Same reasoning as for solids ($\rightarrow$ Sect. 9.2.1)

Conservation of energy:

$$\frac{d}{dt}\int_V \rho u \,\mathrm{d}\boldsymbol{x} + \int_{\partial V} \mathbf{j}\cdot\boldsymbol{n}\,\mathrm{d}S = \int_V f\,\mathrm{d}\boldsymbol{x} \quad \text{for all "control volumes" } V\,. \tag{9.2.1.3}$$

energy stored in V — power flux through ∂V — heat generation in V

In local form:

$$\frac{\partial}{\partial t}(\rho u)(\boldsymbol{x},t) + (\mathrm{div}_{\boldsymbol{x}}\,\mathbf{j})(\boldsymbol{x},t) = f(\boldsymbol{x},t) \quad \text{in } \widetilde{\Omega}\,. \tag{9.2.1.5}$$

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+ Generalized Fourier law:

$$\mathbf{j}(\boldsymbol{x},t) = -\kappa\,\mathbf{grad}\,u(\boldsymbol{x},t) + \mathbf{v}(\boldsymbol{x},t)\rho u(\boldsymbol{x},t)\,,\quad \boldsymbol{x}\in\Omega\,. \tag{10.1.2.4}$$

▷ Transient CDE:

$$\frac{\partial}{\partial t}(\rho u) - \operatorname{div}(\kappa\,\mathbf{grad}\,u) + \operatorname{div}(\rho\mathbf{v}(\boldsymbol{x},t)u) = f(\boldsymbol{x},t) \quad \text{in } \widetilde{\Omega} := \Omega\times]0,T[\,. \tag{10.1.4.1}$$

+ "elliptic" boundary conditions

+ initial condition : $u(x,0) = u_0(x)$

Equivalent for $\operatorname{div}_x \underline{v}(\underline{x},t) = 0$:

$$\boxed{\tfrac{\partial}{\partial t}(\rho u) - \kappa\Delta u + \rho\mathbf{v}(\boldsymbol{x},t)\cdot\mathbf{grad}\,u = f(\boldsymbol{x},t) \quad \text{in } \widetilde{\Omega} := \Omega\times]0,T[}\,. \tag{10.1.4.2}$$

▷ Review questions 10.1.4.4