

ETH Lecture 401-0674-00L Numerical Methods for Partial Differential Equations

~~Numerical Methods for
Partial Differential Equations~~

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(C) Seminar für Angewandte Mathematik, ETH Zürich

Welcome & Introduction

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I Topic

Numerical Methods for **continuum models**
with local interaction**continuum** : unknowns are functions* $u = u(x)$,
 $u = u(x, t)$: ∞ -dimensional spaces
*also fields**models** : from fluid mechanics
electromagnetics
quantum chemistry
quant. finance→ has context, has **structure**PDE $\neq$ just equation $F(u, Du) = 0$
↑
some derivatives
↓
should be processed by numerical method

Example: Heat equation :

$$u = u(x, t) : \frac{\partial u}{\partial t} - \Delta u = f(x, t)$$

→ Dissipative evolution towards equilibrium

(2)

(Stationary, incompressible) **Navier-Stokes equations**:

$$\begin{aligned} -\nu \Delta \mathbf{u} + D\mathbf{u} \cdot \mathbf{u} + \text{grad } p &= \mathbf{f} \quad \text{in } \Omega, \\ \text{div } \mathbf{u} &= 0 \quad \text{in } \Omega, \leftarrow \text{incompressibility} \\ \mathbf{u} &= 0 \quad \text{on } \partial\Omega. \end{aligned}$$

(0.4.3.1)

$\nu \hat{=}$ dynamic viscosity
 $\mathbf{f} : \Omega \mapsto \mathbb{R}^3 \hat{=}$ given external force field
 $\mathbf{u} : \Omega \mapsto \mathbb{R}^3 \hat{=}$ velocity field (unknown)
 $p : \Omega \mapsto \mathbb{R} \hat{=}$ pressure (unknown)

II. Prerequisites $\triangleright$ Sect. 0.3.

III. Contents and Outline

I. Equilibrium models (Elliptic boundary-value problems)

(Ex.: $\Delta u = f$ in $\Omega \subset \mathbb{R}^d + \underbrace{u = 0 \text{ on } \partial\Omega}_{\text{boundary cond.}}$)

- Theory: variational approach
- Finite-element method:
 - Foundations
 - Algorithms
 - Implementation

II. Evolution problem in finite-dimensional state space
(Initial-value problems for ODEs)

$$\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y}) \quad , \quad \mathbf{y}(0) = \mathbf{y}_0 \in \mathbb{R}^n$$

- single-step method
- stability theory

III. Linear evolution problems for fields

- Parabolic initial-boundary-value problems

Ex.: Heat equation

IV. Non-linear conservation laws (in 1D)

$$u = u(x, t) : \frac{\partial u}{\partial t} + \partial_x f(u) = 0$$

- Focus:
- not on theory
 - not on recipes
 - on **algorithms** ($\rightarrow$ coding projects)
 - on behavior / scope / limitations of methods

③

Workload : 300 h = $\underbrace{200 \text{ h}}_{\text{during term}} + \underbrace{100 \text{ h}}_{\text{exam prep.}}$
 $\downarrow$
 (12-15 h / week)

Report errors / make suggestions through Moodle Fora!

Flipped-classroom advice

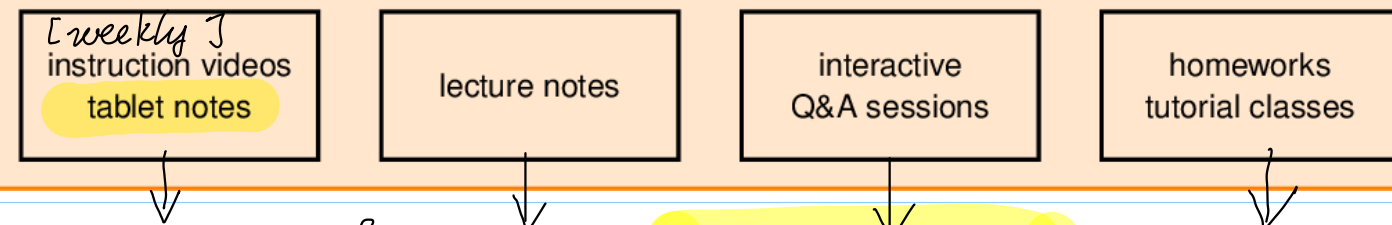
- Develop a routine
- Stable setting
- Take breaks
- Group work

IV. Flipped-Classroom Teaching Format

A flipped-classroom course

This course will follow the **flipped-classroom** paradigm:

Learning by **self-study** guided by



Defines course contents

reference →

ask questions in Moodle Fora

Use pause button!

(3-5 h / week)

(5-7 h / week)

