ETH Lecture 401-0674-00L Numerical Methods for Partial Differential Equations

~~Numerical Methods for Partial Differential Equations~~

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Welcome & Introduction

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I Topic

Numerical Methods for continuum models with local interaction

continuum : unknowns are functions* $u = u(x)$, $u = u(x,t)$: ∞-dimensional spaces
*also fields

models : from fluid mechanics
electromagnetics
quantum chemistry
quant. finance

→ has context, has structure

PDE $\neq$ just equation $F(u, Du) = 0$
(Du: some derivatives)

should be preserved by numerical method

Example: Heat equation :

$$u = u(x,t) : \quad \frac{\partial u}{\partial t} - \Delta u = f(x,t)$$

→ Dissipative evolution towards equilibrium

(Stationary, incompressible) Navier-Stokes equations:

$$\begin{aligned} -\nu\Delta\mathbf{u} + D\mathbf{u}\cdot\mathbf{u} + \mathbf{grad}\,p &= \mathbf{f} \quad \text{in } \Omega\,,\\ \operatorname{div}\mathbf{u} &= 0 \quad \text{in } \Omega\,, \leftarrow \text{incompressibility}\\ \mathbf{u} &= 0 \quad \text{on } \partial\Omega\,. \end{aligned} \tag{0.4.3.1}$$

ν $\hat{=}$ dynamic viscosity
$\mathbf{f}:\Omega\mapsto\mathbb{R}^3$ $\hat{=}$ given external force field
$\mathbf{u}:\Omega\mapsto\mathbb{R}^3$ $\hat{=}$ velocity field (unknown)
$p:\Omega\mapsto\mathbb{R}$ $\hat{=}$ pressure (unknown)

II. Prerequisites ▷ Sect. 0.3.

III. Contents and Outline

I. Equilibrium models (Elliptic boundary-value problems)

(Ex.: $\Delta u = f$ in $\Omega\subset\mathbb{R}^d$ + $\underbrace{u = 0 \text{ on } \partial\Omega}_{\text{boundary cond.}}$)

- Theory: variational approach
- Finite-element method:
  - Foundations
  - Algorithms
  - Implementation

II. Evolution problem in finite-dimensional state space (Initial-value problems for ODEs)

$$\dot{y} = f(y)\,,\quad y(0) = y_0 \in \mathbb{R}^n$$

- single-step method
- stability theory

III. Linear evolution problems for fields

- Parabolic initial-boundary-value problems

Ex: Heat equation

IV. Non-linear conservation laws (in 1D)

$u = u(x,t)$: $\frac{\partial u}{\partial t} + \partial_x f(u) = 0$

Focus:
- not on theory
- not on recipes
- on algorithms (→ Coding projects)
- on behavior / scope / limitations of methods

Workload: $300\,h = \underbrace{200\,h}_{\text{during term}} + \underbrace{100\,h}_{\text{exam prep.}}$

(12-15 h / week)

IV. Flipped-Classroom Teaching Format

A flipped-classroom course

This course will follow the **flipped-classroom** paradigm:

Learning by **self-study** guided by

[weekly] instruction videos tablet notes	lecture notes	interactive Q&A sessions	homeworks tutorial classes

Defines course contents $\xrightarrow{\text{reference}}$

ask questions in Moodle Fora

Use pause button!

(3-5 h / week)

(5-7 h / week)

Report errors / make suggestions through Moodle Fora!

Flipped-classroom advice

- Develop a routine
- Stable setting
- Take breaks
- Group work

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