SOLUTION of (2-3.b):

We can simply rely on

$$\underbrace{\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & 0 \\ \mathbf{A}_{21}\mathbf{A}_{11}^{-1} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ 0 & \mathbf{S} \end{bmatrix}}_{\text{block LU-factorization}} \quad , \quad \text{with Schur complement } \boxed{\mathbf{S} := \mathbf{A}_{22} - \mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{12}} \, .$$

[Lecture → (2.3.2.20)]

Here we have $\mathbf{A}_{11} = \mathbf{R}$, $\mathbf{A}_{12} = \mathbf{v}$, $\mathbf{A}_{21} = \mathbf{u}^\top$, $\mathbf{A}_{22} = \mathbf{O}$. Thus the sought LU-decomposition is

$$\mathbf{A} = \mathbf{L}\mathbf{U} = \begin{bmatrix} \mathbf{I} & 0 \\ \mathbf{u}^\top\mathbf{R}^{-1} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{v} \\ 0 & -\mathbf{u}^\top\mathbf{R}^{-1}\mathbf{v} \end{bmatrix}$$

Here it is important that $\mathbf{R}$ was assumed to be invertible.