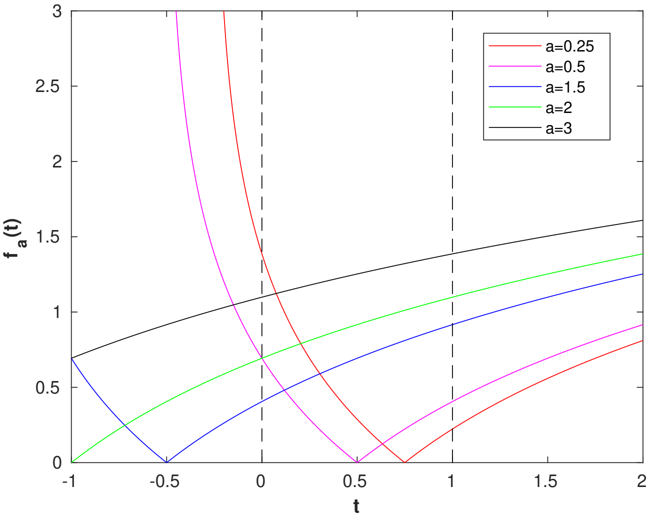
SOLUTION of (6-10.b):



◁ Graphs of $t \mapsto f_a(t)$ for the values of a occurring in the above table.

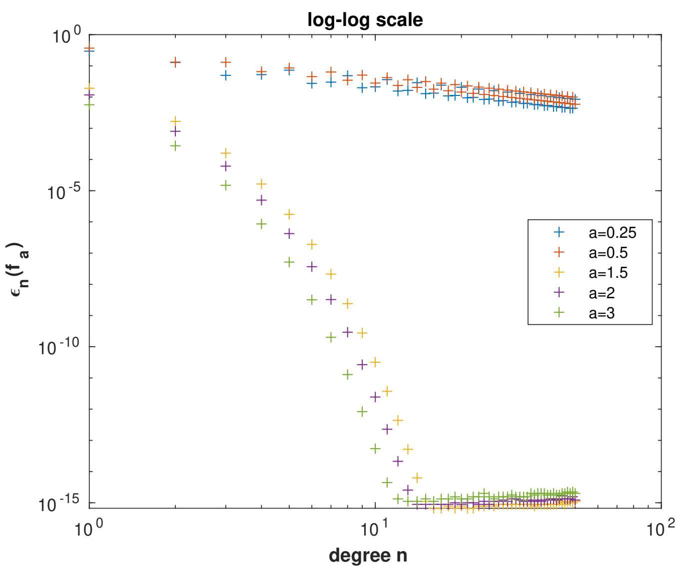
The interval $[0,1]$ is marked with dashed vertical lines.

The function $t \mapsto \log(t+a)$ is defined on $]-a,\infty[$ and changes sign in $t = 1-a$. Hence,

- if $0 < a < 1$, then $f_a(t) = |\log(t+a)|$ has a "kink" in the interval $]0,1[$, a lack of smoothness, that will lead to slow (algebraic) convergence of $\epsilon_n(f_a) \to 0$ as $n \to 0$.
- if $a > 1$, then f_a is *analytic* in $[0,1]$. The nearest $z \in \mathbb{C}$, where analyticity breaks down is $z = 1 - a <$ As a consequence the domain of analyticity is the larger, the larger the value of a; we expect exponential convergence $\epsilon_n(f_a) \to 0$ as $n \to \infty$, which will be faster for larger values of $a > 1$.

  Alternatively, one can resort to the interpolation error estimate [Lecture → (6.2.3.17)] which indicates that the error is proportional to the $(n+1)$-th derivative of f. It is easy to see that $\|f_a^{(n+1)}\|_{\infty,[0,1}$ $(-1)^{n+1}a^{-n-1}$ for $a > 1$. Hence, we expect lower error (faster convergence) for a with larger value.

The following plots display $n \mapsto \epsilon_n(f_a)$ for the values of a listed in the above table.



Doubly logarithmic plot

	exponential convergence	algebraic convergence	rank (exp. cvg. only)
$a = \frac{1}{4}$	—	✔	
$a = \frac{1}{2}$	—	✔	
$a = \frac{3}{2}$	✔	—	3
$a = 2$	✔	—	2
$a = 3$	✔	—	1