

ETH Lectures 401-0663-00L Numerical Methods for Computer Science
401-2673-00L Numerical Methods for CSE

Homework Problems and Projects

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[Link](#) to the current version of this homework collection

0.1 General Information

0.1.1 Weekly Homework Assignments

All problems will be published in this single “.pdf” file. Every week, we publish a list of problem numbers on the lecture Moodle page. These are the current assignments for that week. Details are given on the lecture Moodle page.

0.1.2 Importance of Homework

Homework assignments are **not** mandatory. However, it is **very important** that you constantly exercise with the material you learn. “Solving” the homework assignments one week before the main exam by looking at the solutions will likely result in failure.

We provide hints and solutions from the beginning. It is your responsibility to look at the solutions only if you are stuck with a difficult problem and you tried to find a solution for a sufficient amount of time.

Most homework problems come with an estimate for the time required for its solution. These times are what we expect a bright and well-prepared student to require for that problem during an exam.

0.1.3 Corrections and Grading of Assignments

Homework assignments will not be graded.

However, you may submit your handwritten solutions by handing them to your tutor directly or uploading them via the Moodle upload interface, see the lecture Moodle page for instructions. The assistants will

Problem 7-5: Quadrature plots

In this problem, we will try and deduce the type of quadrature and the integrand from an error plot.

This is a purely theoretical problem discussing convergence of quadrature rules, see [Lecture → Lemma 7.4.3.6], [Lecture → Ex. 7.4.3.9], [Lecture → § 7.5.0.9], [Lecture → Exp. 7.5.0.12].

We consider three different functions on the interval $I = [0, 1]$, which have the following properties:

function A: $f_A \in C^\infty(I)$, $f_A \notin \mathcal{P}_k \forall k \in \mathbb{N}$;

function B: $f_B \in C^0(I)$, $f_B \notin C^1(I)$;

function C: $f_C \in \mathcal{P}_{12}$,

where $\mathcal{P}_k$ is the space of the polynomials of degree at most k defined on I . The following quadrature rules are applied to these functions:

- quadrature rule A is a global Gauss quadrature;
- quadrature rule B is a composite trapezoidal rule;
- quadrature rule C is a composite 2-point Gauss quadrature.

The corresponding absolute values of the quadrature errors are plotted against the number of function evaluations in the figure below. Notice that only the quadrature errors obtained with an even number of function evaluations are shown.

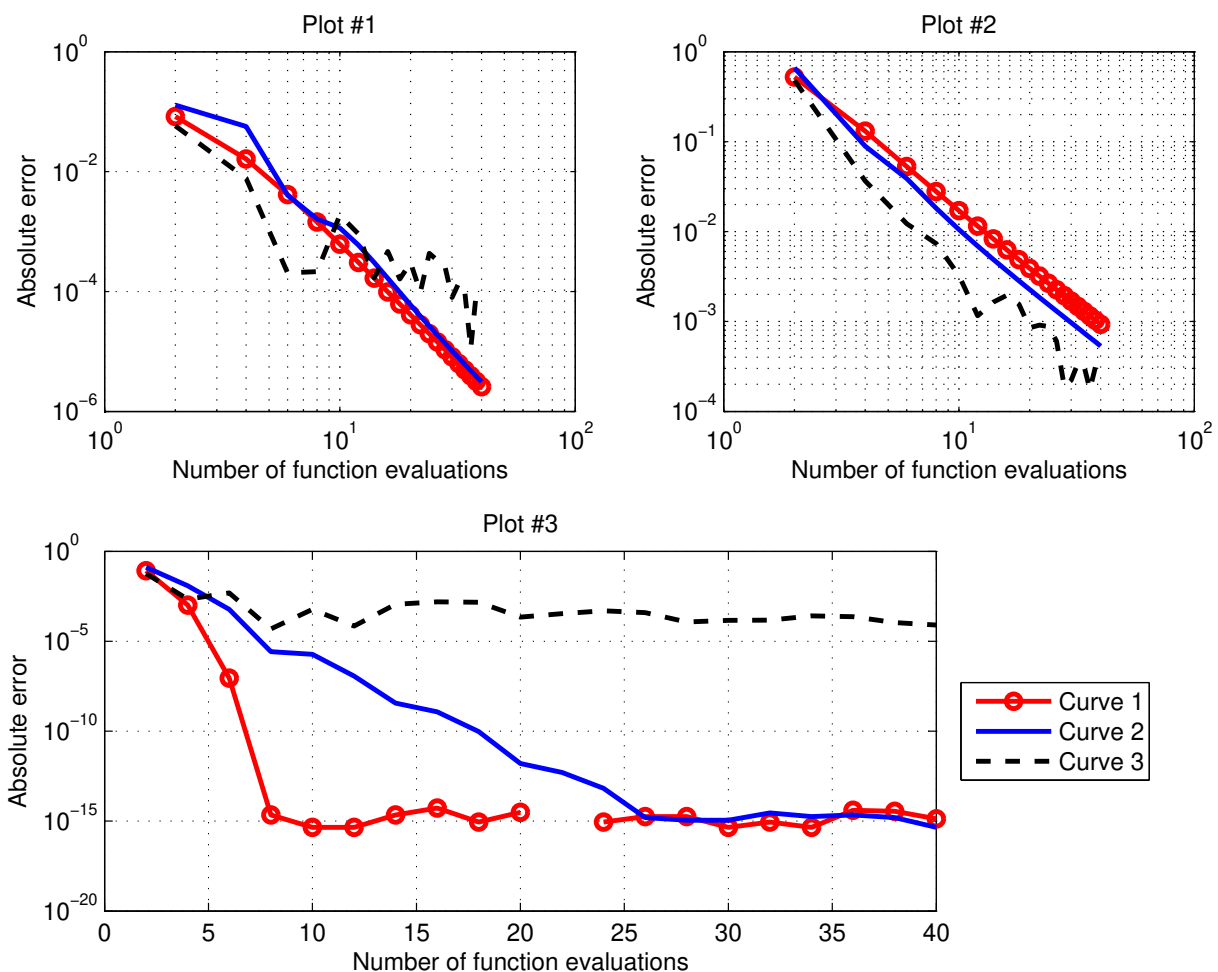


Fig. 63

(7-5.a) ☐ Match the three plots (plot #1, #2 and #3) with the three quadrature rules (quadrature rule A, B, and C). Justify your answer.

HINT 1 for (7-5.a): Notice the different axis scales in the plots.

SOLUTION of (7-5.a):

Plot #1 — Quadrature rule C, Composite 2-point Gauss:
algebraic convergence for every function, about 4^{th} order for two functions.

Plot #2 — Quadrature rule B, Composite trapezoidal:
algebraic convergence for every function, about 2^{nd} order.

Plot #3 — Quadrature rule A, Global Gauss:
algebraic convergence for one function, exponential for another one, exact integration with 8 evaluations for the third one.

(7-5.b)  The quadrature error curves for a particular function f_A , f_B and f_C are plotted in the same style (curve 1 as red line with small circles, curve 2 means the blue solid line, curve 3 is the black dashed line). Which curve corresponds to which function (f_A , f_B , f_C)? Justify your answer.

SOLUTION of (7-5.b):

Curve 1 red line and small circles — f_C polynomial of degree 12:
integrated exactly with 8 evaluations with global Gauss quadrature.

Curve 2 blue continuous line only — f_A analytic function:
exponential convergence with global Gauss quadrature.

Curve 3 black dashed line — f_B non smooth function:
algebraic convergence with global Gauss quadrature.

End Problem 7-5