ETH Lectures 401-0663-00L Numerical Methods for Computer Science
401-2673-00L Numerical Methods for CSE

Homework Problems and Projects

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Link to the current version of this homework collection

0.1 General Information

0.1.1 Weekly Homework Assignments

All problems will be published in this single ".pdf" file. Every week, we publish a list of problem numbers on the lecture Moodle page. These are the current assignments for that week. Details are given on the lecture Moodle page.

0.1.2 Importance of Homework

Homework assignments are **not** mandatory. However, it is **very important** that you constantly exercise with the material you learn. "Solving" the homework assignments one week before the main exam by looking at the solutions will likely result in failure.

We provide hints and solutions from the beginning. It is your responsibility to look at the solutions only if you are stuck with a difficult problem and you tried to find a solution for a sufficient amount of time.

Most homework problems come with an estimate for the time required for its solution. These times are what we expect a bright and well-prepared student to require for that problem during an exam.

0.1.3 Corrections and Grading of Assignments

Homework assignments will not be graded.

However, you may submit your handwritten solutions by handing them to your tutor directly or uploading them via the Moodle upload interface, see the lecture Moodle page for instructions. The assistants will

Chapter 8

Iterative Methods for Non-Linear Systems of Equations

Problem 8-1: Convergent Newton iteration

As explained in [Lecture → Section 8.4.2.1], the convergence of Newton's method in 1D may only be local. This problem investigates a particular setting, in which global convergence can be expected.

This is a purely theoretical problem practising "intuitive mathematical reasoning".

We recall the notion of a *convex function* [Lecture → Def. 5.3.1.4] and its geometric meaning [Lecture → Fig. 157]: A differentiable function $f : [a,b] \mapsto \mathbb{R}$ is convex if and only if its graph lies on or above its tangent at any point. Equivalently, differentiable function $f : [a,b] \mapsto \mathbb{R}$ is convex, if and only if its derivative is non-decreasing.

(8-1.a) ⚂ (20 min.) Give a "graphical proof" of the following statement:

Theorem 8.1.1. Global convergence of Newton's method in 1D for convex monotone functions

If $F(x)$ belongs to $C^2(\mathbb{R})$, is strictly increasing, is convex, and has a unique zero, then the Newton iteration [Lecture → Eq. (8.4.2.1)] for $F(x) = 0$ is well defined and will converge to the zero of $F(x)$ for any initial guess $x^{(0)} \in \mathbb{R}$.

SOLUTION of (8-1.a):

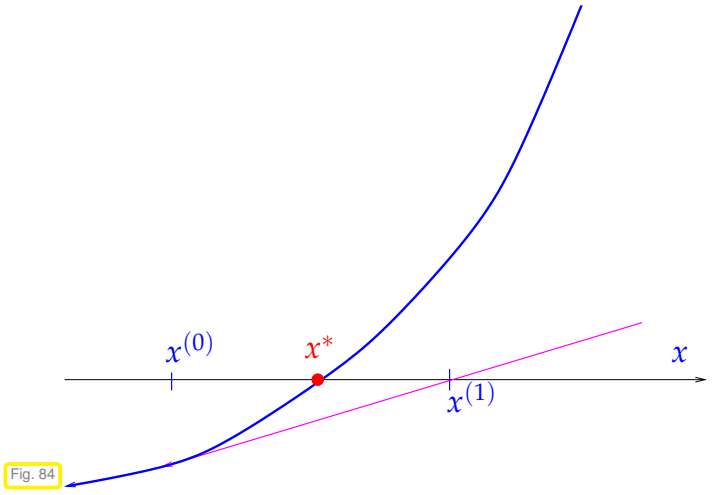


Fig. 84

If $x^{(0)} < x^*$, where $F(x^*) = 0$, then $x^{(1)} > x^*$ and $F(x^{(1)}) > 0$.
Therefore, we may assume $x^{(1)} > x^*$ from now.

Assume $x^{(k)} > x^*$, which means $F(x^{(k)}) > 0$ thanks to the monotonicity of F.

- Since f is *convex* the tangent at the graph of f in $x^{(k)}$ is below the graph. This implies that its intersection with the x-axis lies to the right of x^*. We conclude $x^{(k+1)} > x^*$
- Since F is monotone, F' is positive. Hence

$$x^{(k+1)} := x^{(k)} - \frac{F(x^{(k)})}{F'(x^{(k)})} < x^{(k)} .$$

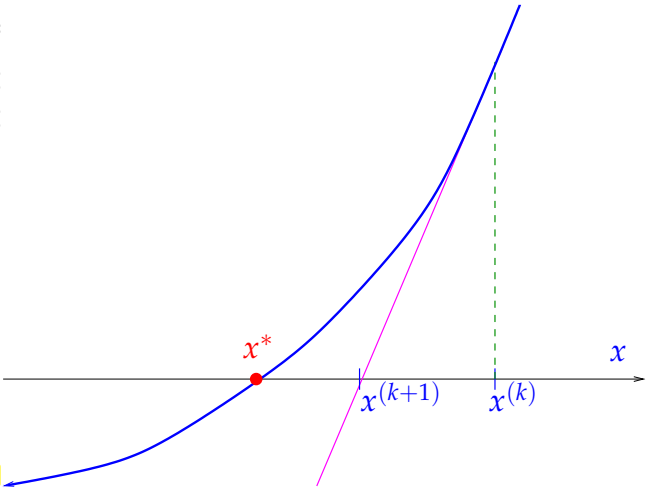


Fig. 85

Therefore, the sequence $\left(x^{(k)}\right)_k$ is decreasing and bounded from below by x^*.
By the completeness of $\mathbb{R}$ such a sequence will have a limit $x^+ := \lim_{k\to\infty} x^{(k)}$.

By the continuity of both F and F', we can simply plug the limit x^+ into the Newton iteration:

$$x^+ = x^+ - \frac{F(x^+)}{F'(x^+)} \quad \blacktriangleright \quad F(x^+) = 0 .$$

▲

End Problem 8-1 , (25 min.)