

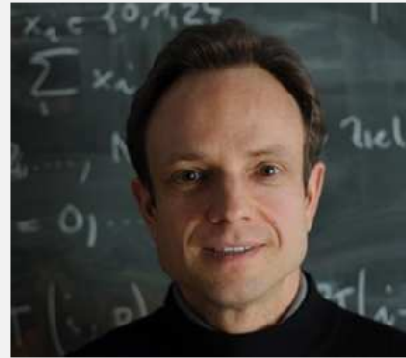
Course Video

Section 1.1.1–Section 1.1.2: Notations and Classes of Matrices

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(C) Seminar für Angewandte Mathematik, ETH Zürich



Prerequisites.

- Concept of “Matrix” and “Vector” from linear algebra.

Dependencies. None

Duration: 10 minutes

I. Computing with Matrices and Vectors

1.1. Fundamentals

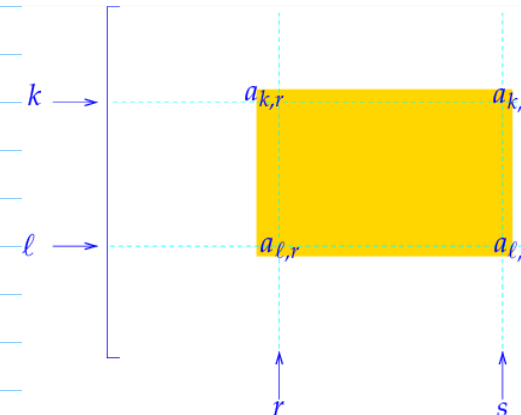
1.1.1. Notations

Vector: $\underline{x} \in \mathbb{K}^n$ ($\mathbb{K} = \mathbb{R}, \mathbb{C}$) $(\underline{x})_i \triangleq i\text{-th component of } \underline{x}, i=1, \dots, n$ Matrix: $A \in \mathbb{K}^{n,m}$

$$A := \begin{bmatrix} a_{11} & \dots & a_{1m} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nm} \end{bmatrix}$$

→ matrix entry/matrix element $(A)_{i,j} := a_{ij}, 1 \leq i \leq n, 1 \leq j \leq m,$ → i -th row, $1 \leq i \leq n$: $(A)_{i,:} := [a_{i,1}, \dots, a_{i,m}]$,→ j -th column, $1 \leq j \leq m$: $(A)_{:,j} := [a_{1,j}, \dots, a_{n,j}]^T$,→ matrix block (sub-matrix) $(A)_{k:\ell, r:s} := [a_{ij}]_{\substack{i=k,\dots,\ell \\ j=r,\dots,s}}, 1 \leq k \leq \ell \leq n, 1 \leq r \leq s \leq m.$

$$\in \mathbb{K}^{\ell-k+1, s-r+1}$$



∇ Linear algebra indexing
starting from 1

VS C++/Python indexing
starting from 0!

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1.1.2. Classes of Matrices

Identity matrix: $\mathbf{I} := \mathbf{I}_n := \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix} \in \mathbb{K}^{n,n},$

Zero matrix: $\mathbf{O} := \mathbf{O}_{n,m} := \begin{bmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix} \in \mathbb{K}^{n,m},$

Diagonal matrix: $\text{diag}(d_1, \dots, d_n) := \begin{bmatrix} d_1 & & 0 \\ & \ddots & \\ 0 & & d_n \end{bmatrix} \in \mathbb{K}^{n,n}, \quad d_j \in \mathbb{K}, \quad j = 1, \dots, n.$

▷ Do review questions 1.1.2.9

Review questions 1.2.9

(Try to answer without consulting notes)

A:

Give a compact notation for the row vector containing the diagonal entries of a square matrix $\mathbf{S} \in \mathbb{R}^{n,n}$, $n \in \mathbb{N}$.

B:

How can you write down the $s \times s$ -submatrix, $s \in \mathbb{N}$, in the upper right corner of $\mathbf{C} \in \mathbb{R}^{n,m}$, $n, m \geq s$.

C:

We consider two matrices $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{n,m}$, both with at most $N \in \mathbb{N}$ non-zero entries. What is the maximal number of non-zero entries of $\mathbf{A} + \mathbf{B}$?

D:

A matrix $\mathbf{A} \in \mathbb{R}^{n,m}$ enjoys the following property (**banded matrix**):

$$i \in \{1, \dots, n\}, j \in \{1, \dots, m\}, i - j \notin \{-B_-, \dots, B_+\} \Rightarrow (\mathbf{A})_{ij} = 0,$$

for given $B_-, B_+ \in \mathbb{N}_0$. What is the maximal number of non-zero entries of $\mathbf{A}$.

E:

A matrix $\mathbf{A}$ with real entries is known to be **skew-symmetric**: $\mathbf{A}^\top = -\mathbf{A}$. What does this tell us about $\mathbf{A}$ and its entries?

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