

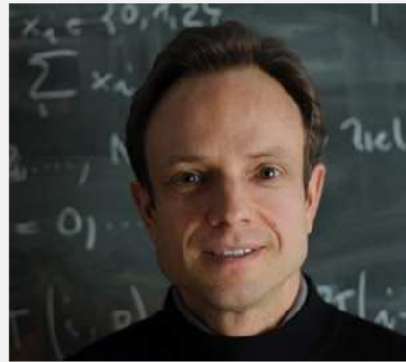
Course Video

Section 2.1–Section 2.2.1: Introduction and Theory: Linear Systems of Equations (LSEs)

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(C) Seminar für Angewandte Mathematik, ETH Zürich



Prerequisites.

- Matric vector calculus
- Notions and facts about linear systems of equations as they were taught in the introductory linear algebra class.

Duration: minutes

II. Direct Methods for Square Linear Systems of Equations

2.1. Introduction:
Linear Systems of Equations (LSE)

LSE : $Ax = b$

$A \in \mathbb{K}^{n,n}$ *square system / coefficient matrix* $b \in \mathbb{K}^n$ *right-hand-side vector*

$\# \text{equation} = \# \text{unknown} = n$

$$\hookrightarrow (A)_{i,:}^T x = (b)_i, \quad i = 1, \dots, n$$

[A scalar equation for each row of A]

Data/Input : A, b Sought/output : x

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2.2. Square LSE : Theory $\rightarrow$ Linear algebra

2.2.1. Existence and Uniqueness of Solutions

Definition 2.2.1.1. Invertible matrix $\rightarrow$

$$A \in \mathbb{K}^{n,n} \text{ invertible/regular} :\Leftrightarrow \exists_! B \in \mathbb{K}^{n,n}: AB = BA = I.$$

B is called the **inverse** of A , (notation $B = A^{-1}$)

A invertible : $A\underline{x} = \underline{b} \Leftrightarrow \underline{x} = A^{-1}\underline{b}$ unique

⚠ Do not use matrix inversion function to solve LSE with numerical libraries

Criteria:

$$A \in \mathbb{K}^{n,n} \text{ regular} \Leftrightarrow \det A \neq 0$$

$$\Leftrightarrow \text{cols lin. indep.}$$

$$\Leftrightarrow \text{rows lin. indep.}$$

$$\Leftrightarrow \mathcal{N}(A) = \{\underline{z} \in \mathbb{K}^n : A\underline{z} = \underline{0}\} = \{\underline{0}\}$$

$\uparrow$
nullspace of A

③

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⑦