

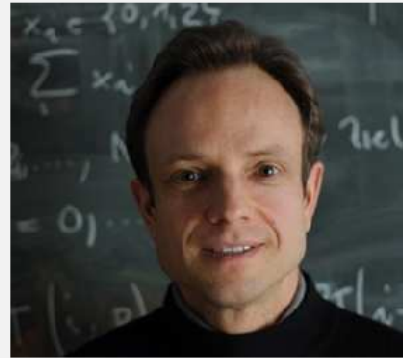
Course Video

Section 4.1.3: Discrete Convolutions

Prof. R. Hiptmair, SAM, ETH Zurich

Date: October 4, 2020

(C) Seminar für Angewandte Mathematik, ETH Zürich



Dependencies. Section 4.1.1, Section 4.1.2

Duration: 7 minutes

Note: Possible minor *mismatch of video and tablet notes!*

[Corrections and updates can be incorporated into tablet notes only]



IV. Filtering Algorithms

4.1. Filters and Convolutions

4.1.3. Discrete Convolutions

LT-FIR channel F , with $\mathbb{R} = (\dots, 0, h_0, h_1, \dots, h_{n-1}, 0, \dots)$

$$\begin{bmatrix} \vdots \\ 0 \\ y_0 \\ y_1 \\ \vdots \\ y_n \\ \vdots \\ \vdots \\ y_{m+n-3} \\ y_{m+n-2} \\ 0 \\ \vdots \end{bmatrix} = x_0 \begin{bmatrix} \vdots \\ 0 \\ h_0 \\ \vdots \\ h_{n-1} \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \\ \vdots \end{bmatrix} + x_1 \begin{bmatrix} \vdots \\ 0 \\ 0 \\ h_0 \\ \vdots \\ h_{n-1} \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \\ \vdots \end{bmatrix} + x_2 \begin{bmatrix} \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ h_0 \\ \vdots \\ \vdots \\ 0 \\ 0 \\ \vdots \end{bmatrix} + \dots + x_{m-1} \begin{bmatrix} \vdots \\ 0 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \\ h_0 \\ \vdots \\ h_{n-1} \\ 0 \\ \vdots \end{bmatrix} \quad (4.1.2.3)$$

② LT-FIR action for $m = 4, n = 5$

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{bmatrix} = x_0 \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_1 \begin{bmatrix} 0 \\ h_0 \\ h_1 \\ h_2 \\ h_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix}$$

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{bmatrix} = \begin{bmatrix} h_0 & 0 & 0 & 0 \\ h_1 & h_0 & 0 & 0 \\ h_2 & h_1 & h_0 & 0 \\ h_3 & h_2 & h_1 & h_0 \\ h_4 & h_3 & h_2 & h_1 \\ 0 & h_4 & h_3 & h_2 \\ 0 & 0 & h_4 & h_3 \\ 0 & 0 & 0 & h_4 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_4 \end{bmatrix}$$

$\boxed{\curvearrowright} \hat{=} \text{cyclic permutation}$

$$\begin{bmatrix} \vdots \\ 0 \\ y_0 \\ y_1 \\ \vdots \\ y_n \\ \vdots \\ y_{m+n-3} \\ y_{m+n-2} \\ 0 \\ \vdots \end{bmatrix} = x_0 \begin{bmatrix} \vdots \\ 0 \\ h_0 \\ \vdots \\ h_{n-1} \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} + x_1 \begin{bmatrix} \vdots \\ 0 \\ 0 \\ \vdots \\ h_{n-1} \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ h_0 \\ h_{n-1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \dots + x_{m-1} \begin{bmatrix} \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ h_0 \\ \vdots \\ h_{n-1} \\ 0 \\ \vdots \end{bmatrix} \quad (4.1.2.3)$$

$\nabla \quad (m \geq n)$

$$\mathbf{y} := \begin{bmatrix} y_0 \\ \vdots \\ y_{m+n-2} \end{bmatrix} = \begin{bmatrix} h_0 & 0 & \dots & 0 \\ h_1 & h_0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_{n-1} & h_{n-2} & \dots & h_0 \\ 0 & h_{n-1} & \dots & h_1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & h_{n-1} \end{bmatrix} \begin{bmatrix} x_0 \\ \vdots \\ x_{m-1} \end{bmatrix} =: \mathbf{C} \mathbf{x} \quad (4.1.3.2)$$

$m+n-1$ (height), m (width)

4

5

7