

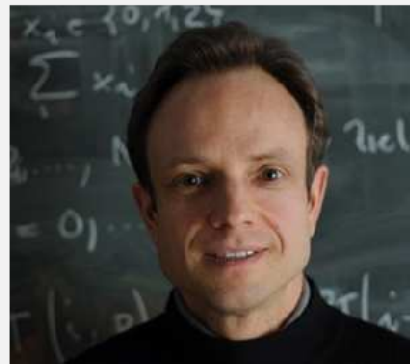
Course Video

Section 5.2.2: Polynomial Interpolation: Theory

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(C) Seminar für Angewandte Mathematik, ETH Zürich



Prerequisites.

- Theory of linear systems of equations from linear algebra

Dependencies. Section 5.2.1

Duration: 10 minutes

Note: Possible minor *mismatch of video and tablet notes!*

[Corrections and updates can be incorporated into tablet notes only]

V. Data Interpolation and Data Fitting in 1D

5.2. Global Polynomial Interpolation

5.2.2. Polynomial Interpolation: Theory

One-dimensional interpolation

Given: data points (t_i, y_i) , $i = 0, \dots, n$, $n \in \mathbb{N}$, $t_i \in I \subset \mathbb{R}$, $y_i \in \mathbb{R}$ Objective: Reconstruction of a function $f : I \mapsto \mathbb{R}$

- satisfying the $n + 1$ **interpolation conditions**

$$f(t_i) = y_i, \quad i = 0, \dots, n. \quad (5.1.0.2)$$

- and belonging to a set V of eligible functions.

The function f we find is called the **interpolant** of the given data set $\{(t_i, y_i)\}_{i=0}^n$. $V = \mathcal{P}_n$:

Lagrange polynomial interpolation problem

Given the set of **interpolation nodes** $\{t_0, \dots, t_n\} \subset \mathbb{R}$, $n \in \mathbb{N}$, and the values $y_0, \dots, y_n \in \mathbb{R}$ compute $p \in \mathcal{P}_n$ such that it satisfies the **interpolation conditions** (IC)

$$p(t_j) = y_j \quad \text{for } j = 0, \dots, n. \quad (5.2.2.2)$$

②

Tool: A **cardinal basis** $\{L_i\}_{i=0}^n$ of $\mathcal{P}_n$

$$L_i(t_j) = \delta_{ij}, \quad i, j \in \{0, \dots, n\}$$

$$L_i(t) = \frac{(t-t_0) \cdots (t-t_{i-1})(t-t_{i+1}) \cdots (t-t_n)}{(t_i-t_0) \cdots (t_i-t_{i-1})(t_i-t_{i+1}) \cdots (t_i-t_n)}$$

$\hookrightarrow$ **Lagrange polynomial** $\in \mathcal{P}_n$

$$\Rightarrow \text{interpolant : } p(t) = \sum_{i=0}^n y_i L_i(t)$$

$$\Rightarrow \begin{cases} \mathcal{P}_n \xrightarrow{\dim=0} \mathbb{R}^{n+1} \\ p \mapsto [p(t_j)]_{j=0}^n \end{cases} \text{ is surjective} \\ \downarrow \\ \text{Uniqueness}$$

$\triangleright$ Review questions 5.2.2.19
(Please answer without aids)

Theorem 5.2.2.7. Existence & uniqueness of Lagrange interpolation polynomial $\rightarrow$,

The general Lagrange polynomial interpolation problem admits a **unique** solution $p \in \mathcal{P}_n$ for **any** (t_i, y_i) .

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4

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7