

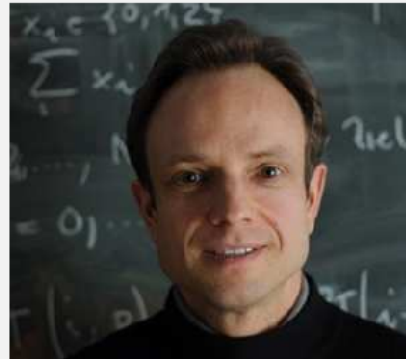
Course Video

Section 5.2.1: Uni-Variate Polynomials

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(C) Seminar für Angewandte Mathematik, ETH Zürich



Prerequisites.

- Concept of a basis from linear algebra

Duration: 10 minutes

V. Data Interpolation and Data Fitting in 1D

5.2. Global Polynomial Interpolation

5.2.1. Uni-Variate Polynomials

Notation: Vector space of the **(uni-variate) polynomials** of **degree** $\leq k$, $k \in \mathbb{N}$:

Now $V := \mathcal{P}_k := \{t \mapsto \alpha_k t^k + \alpha_{k-1} t^{k-1} + \dots + \alpha_1 t + \alpha_0 \cdot 1, \alpha_j \in \mathbb{R}\}.$ (5.2.1.1)

$\nwarrow$ leading coefficient $\downarrow$ monomial representation



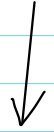
$$\{t \mapsto t^l\}_{l=0}^k \stackrel{!}{=} \text{monomial basis of } \mathcal{P}_k$$

$$\Rightarrow \dim \mathcal{P}_k = k+1$$

(2)

§ 5.2.1.3 : The charms of polynomials

- Vector space & $\text{pol.} * \text{pol.} \rightarrow \text{pol.}$
- easy to derive & integrate
- Taylor polys. $\Rightarrow$ approximation of fcts.
- easy to evaluate



§ 5.2.1.5. Horner scheme

$\rightarrow p \in \mathcal{P}_n$ in monomial representation
 $p(t)$ with cost = $O(n)$

$$p(t) = t(\cdots t(\underbrace{t(\alpha_n t + \alpha_{n-1}) + \alpha_{n-2}}_{\cdots} + \alpha_1) + \alpha_0). \quad (5.2.1.6)$$

C++-code 5.2.1.7: Horner scheme (vectorized version)

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2 // Efficient evaluation of a polynomial in monomial representation
3 // using the Horner scheme (5.2.1.6)
4 // IN: p = vector of monomial coefficients, length = degree + 1
5 // (leading coefficient in p(0), PYTHON convention Rem. 5.2.1.4)
6 // t = vector of evaluation points t_i
7 // OUT: vector of values: polynomial evaluated at t_i
8 Eigen::VectorXd horner(const Eigen::VectorXd &p, const Eigen::VectorXd &t) {
9     const VectorXd::Index n = t.size();
10    Eigen::VectorXd y{p[0] * VectorXd::Ones(n)};
11    for (unsigned i = 1; i < p.size(); ++i)
12        y = t.cwiseProduct(y) + p[i] * VectorXd::Ones(n);
13    return y;
14 }

```

Convention: $p = [\alpha_n \ \alpha_{n-1} \ \dots \ \alpha_0]^T \in \mathbb{R}^{n+1}$

▷ Review questions 5.2.1.8

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