

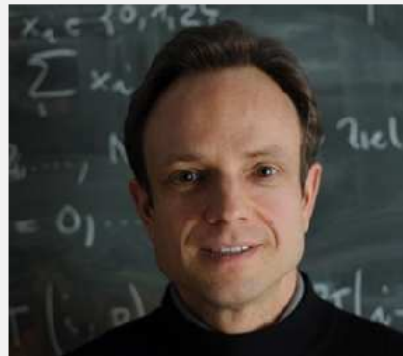
Course Video

Section 5.4.1: Spline Function Spaces

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(C) Seminar für Angewandte Mathematik, ETH Zürich



Dependencies. Section 5.2.1

Duration: 5 minutes

V. Data Interpolation and Data Fitting in 1D

5.4 Splines

5.4.1. Spline Function Spaces

Definition 5.4.1.1. Spline space $\rightarrow$

Given an interval $I := [a, b] \subset \mathbb{R}$ and a **knot sequence** $\mathcal{M} := \{a = t_0 < t_1 < \dots < t_{n-1} < t_n = b\}$, $n \in \mathbb{N}$, the vector space $\mathcal{S}_{d, \mathcal{M}}$ of the **spline functions** of degree d (or order $d+1$) is defined by

$$\mathcal{S}_{d, \mathcal{M}} := \{s \in C^{d-1}(I) : s_j := s|_{[t_{j-1}, t_j]} \in \mathcal{P}_d \forall j = 1, \dots, n\}.$$

 $d-1$ -times continuously differentiablelocally polynomial of degree d

⚠ [for interpolation]: knots $\neq$ node

(2)

Examples:

 $d = 0$: p.w. constant ("C⁻¹ $\hat{=}$ discontinuous") $d = 1$: C⁰ & p.w. linear (Graph $\hat{=}$ polygon) $d = 2$: C¹ & p.w. quadraticIssue : $S_{d,M} \neq \mathcal{P}_d$?dim $S_{d,M}$ by a counting argument

$$\dim S_{d,M} = \underbrace{n \cdot \dim \mathcal{P}_d}_{\substack{\text{\# intervals} \\ = d+1}} - \underbrace{\#\{\text{continuity constraints}\}}_{\substack{= (n-1)d \\ \uparrow \\ \text{no. of interior knots}}}$$

Theorem 5.4.1.2. Dimension of spline spaceThe space $S_{d,M}$ from Def. 5.4.1.1 has dimension

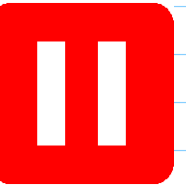
$$\dim S_{d,M} = n + d.$$

$$\dim S_{d,M} > \dim \mathcal{P}_d \quad \text{if } n \geq 2$$

Rem 5.4.1.3 : (Differentiating and Integrating Splines)

$$s \in S_{d,M} \Rightarrow s' \in S_{d-1,M}$$

$$\Rightarrow \{t \rightarrow \int_a^t s(\tau) d\tau\} \in S_{d+1,M}$$



▷ Review questions 5.4.1.3

③

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