

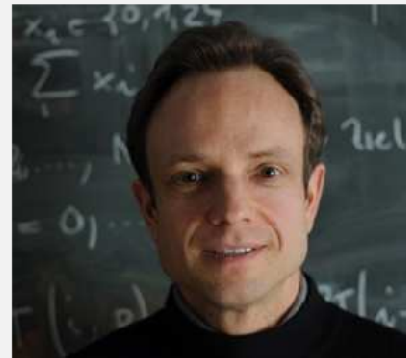
Course Video

Section 6.1: Approximation of Functions in 1D:
Introduction

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(C) Seminar für Angewandte Mathematik, ETH Zürich



Prerequisites.

- Functions and norms
- Object-oriented programming

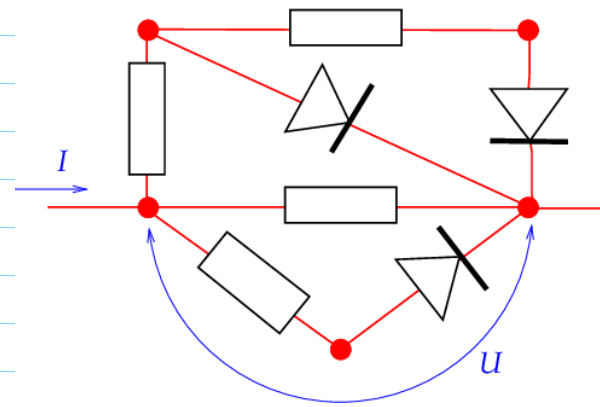
Dependencies. Section 5.1

Duration: 13 minutes

VI. Approximation of Functions
in One Dimension

6.1. Introduction

Ex 6.1.0.3 (Model reduction in circuit simulation)

◁ 2-port circuit element,
described by $I = I(u)$ $I(u)$ is computable $\forall u$,
but this is *expensive*Idea: replace $u \mapsto I(u)$ with $u \mapsto \tilde{I}(u)$

- that
- allows fast evaluation
 - is "close to" $I(u)$

$\tilde{I} \approx$ *surrogate function*
[Model reduction]

② Implementation: Functor-type class

C++ code 6.1.0.4: Class describing a 2-port circuit element for circuit simulation

```

1 class CircuitElement {
2   private:
3     // internal data describing  $U \mapsto \tilde{I}(U)$ .
4   public:
5     // Constructor taking some parameters and building  $\tilde{I}$ 
6     CircuitElement(const Parameters &P);
7     // Point evaluation operators for  $\tilde{I}$  and  $\frac{d}{dU}\tilde{I}$ 
8     double I(double U) const;
9     double dIdU(double U) const;
10 };

```

§ 6.1.0.1 General Function Approximation Problem

only point evaluation possible

Approximation of functions: Generic view

Given: a **function** $f : D \subset \mathbb{R}^n \mapsto \mathbb{R}^d$, $n, d \in \mathbb{N}$, often in **procedural form**, e.g. for $n = d = 1$, as **double f(double)** ($\rightarrow$ Rem. 5.1.0.9).

Goal: Find a **"simple"** function $\tilde{f} : D \mapsto \mathbb{R}^d$ such that the **approximation error** $f - \tilde{f}$ is **"small"**.

(p.w.) polynomials

Needed: norms on function spaces

- **Supremum norm** : $\|g\|_{\infty} := \sup_{t \in D} |g(t)|$
- **L^2 -norm** : $\|g\|_2 := \int_D |g(t)|^2 dt$

Policy: Approximation by interpolation

Interpolation scheme + sampling $\rightarrow$ approximation scheme

$$f : I \subset \mathbb{R} \rightarrow \mathbb{K} \xrightarrow{\text{sampling}} (t_i, y_i := f(t_i))_{i=0}^m \xrightarrow{\text{interpolation}} \tilde{f} := \mathcal{I}_{\mathcal{T}} y \quad (\tilde{f}(t_i) = y_i).$$

free choice of nodes $t_i \in I$: NEW!

Also now: notion of approximation error

