

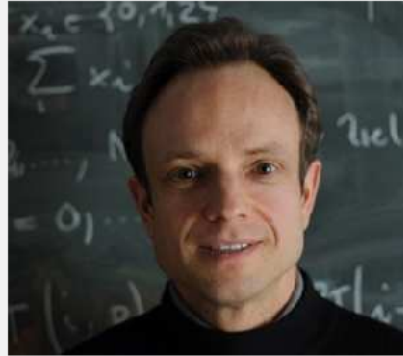
Course Video

Section 6.5.1: Approximation by Trigonometric Interpolation

Prof. R. Hiptmair, SAM, ETH Zurich

Date: November 15, 2020

(C) Seminar für Angewandte Mathematik, ETH Zürich



Dependencies. Section 5.6

Duration: 7 minutes

VI. Approximation of Functions
in 1D

6.5 Approximation by Trigonometric Polynomials

$f: \mathbb{R} \rightarrow \mathbb{R}$, 1 -periodic, continuous : $f(t+1) = f(t)$
 $\mathbb{C}$ $\rightarrow$ also for approximation
 $\rightarrow$ use trigonometric polynomial

$$\mathcal{P}_{2n}^T = \text{Span} \{ 1, \sin(2\pi t), \cos(2\pi t), \sin(4\pi t), \cos(4\pi t), \dots, \sin(2\pi n t), \cos(2\pi n t) \}$$

$$= \text{Span} \{ t \mapsto \exp(-2\pi i k t) \}_{k=-n}^n$$

degree $2n$ $\uparrow$

$$\dim \mathcal{P}_{2n}^T = 2n + 1$$

②

6.5.1. Approximation by Trigonometric Interpolation

Interpolation scheme + sampling $\rightarrow$ approximation scheme

$$f: I \subset \mathbb{R} \rightarrow \mathbb{K} \xrightarrow{\text{sampling}} (t_i, y_i := f(t_i))_{i=0}^m \xrightarrow{\text{interpolation}} \tilde{f} := \mathcal{I}_{\mathcal{T}} \mathbf{y} \quad (\tilde{f}(t_i) = y_i) .$$

$\uparrow$
 free choice of nodes $t_i \in I$

(Trigonometric)

Recall: Trigonometric interpolation $\rightarrow$ Section 5.6

Given nodes $t_0 < t_1 < \dots < t_{2n}$, $t_k \in [0, 1[$, and values $y_k \in \mathbb{R}$, $k = 0, \dots, 2n$ find

$$q \in \mathcal{P}_{2n}^T := \text{Span}\{t \mapsto \cos(2\pi jt), t \mapsto \sin(2\pi jt)\}_{j=0}^n, \quad (6.5.1.2)$$

$$\text{with } q(t_k) = y_k \text{ for all } k = 0, \dots, 2n. \quad (6.5.1.3)$$

Terminology: $\mathcal{P}_{2n}^T \triangleq$ space of **trigonometric polynomials** of degree $2n$.

exists / uniqueness

Note: **Equidistant node** $\Rightarrow$ FFT-based algorithms

