

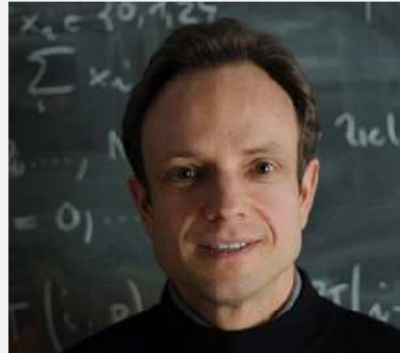
ETH Lecture 401-0663-00L Numerical Methods for CSE

Course Video Section 7.2: Quadrature Formulas/Rules

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(C) Seminar für Angewandte Mathematik, ETH Zürich



Prerequisites.

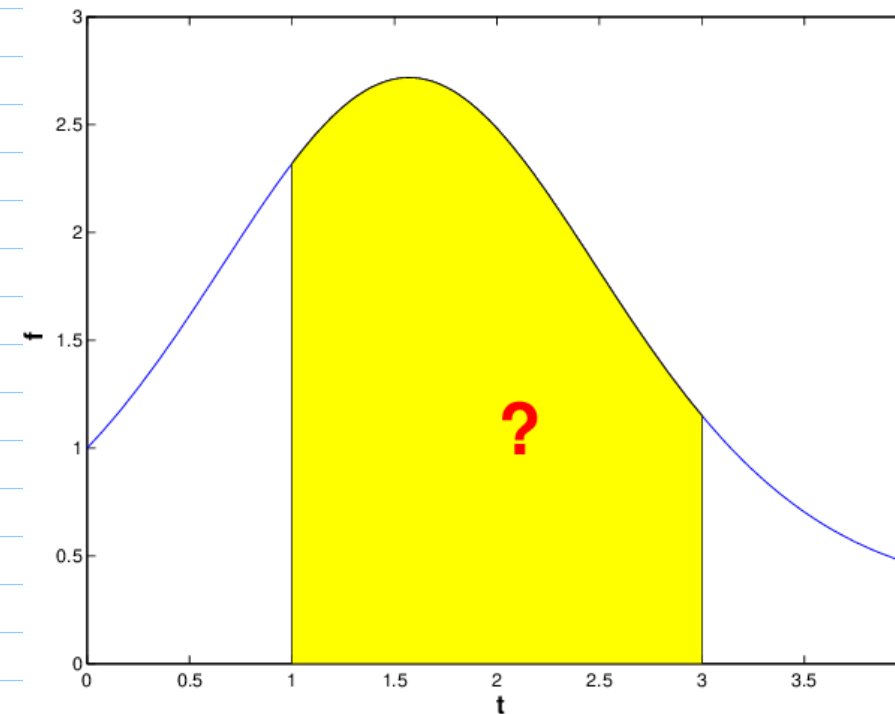
- Substitution/transformation formula for integrals

Dependencies. Section 6.1

Duration: 25 minutes

VII Numerical Quadrature

7.2. Quadrature Formulas/Rules



Task :

Approximate

$$I := \int_a^b f(t) dt$$

f given in procedural form

`double f(double)`

$$J := \{t_1, \dots, t_n\} \stackrel{\wedge}{=} \text{interp. node}, t_z \in I$$

I.C. $(I_S \gamma)(t_j) = (\gamma)_j, j = 1, \dots, n$

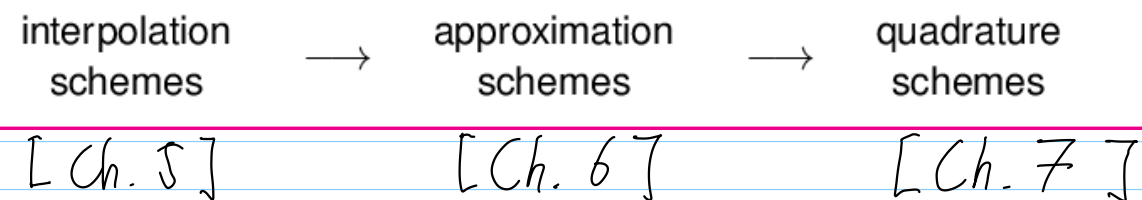
Assume : I_S is linear

$$= \sum_{j=1}^n f(t_j) \underbrace{\int_1^b I_S(\vec{e}_j)(\tau) d\tau}_{\text{weight } w_j}$$

I_S is linear

quad. nodes

cardinal function
(e.g. Lagrange polynomials)



Focus : families of Q.R. , n -dependence of quadrature error $E_n(f) = \left| \int_a^b f(t) dt - Q_n(f) \right|$

asymptotics for $n \rightarrow \infty$

Q.R. from interpolation scheme $I_{\mathcal{T}}$

$$\begin{aligned} E_n(f) &= \left| \int_a^b f(t) dt - \int_a^b I_J([f(t_j)]_{j=1}^n)(t) dt \right| \\ &\leq |b-a| \|f - I_J([f(t_j)]_{j=1}^n)\|_{\infty, [a,b]} \end{aligned}$$

max. norm of interpolation error

