

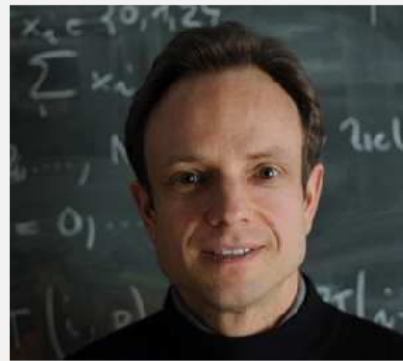
Course Video

Section 7.3: Polynomial Quadrature Formulas

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Dependencies. Section 5.2

Duration: 16 minutes

VII Numerical Quadrature

7.3 Polynomial Quadrature Formulas

Recall: Q.R. by interpolation

$$\int_a^b f(t) dt \approx \int_a^b l_{\mathcal{T}}[f(t_0), \dots, f(t_n)]^T dt = \sum_{j=0}^n f(t_j) \underbrace{\int_a^b (l_{\mathcal{T}}(\mathbf{e}_j))(t) dt}_{\text{weight } w_j}. \quad (7.2.0.11)$$

↑
interpolation scheme based on node set $\mathcal{T} = \{t_0, \dots, t_n\}$

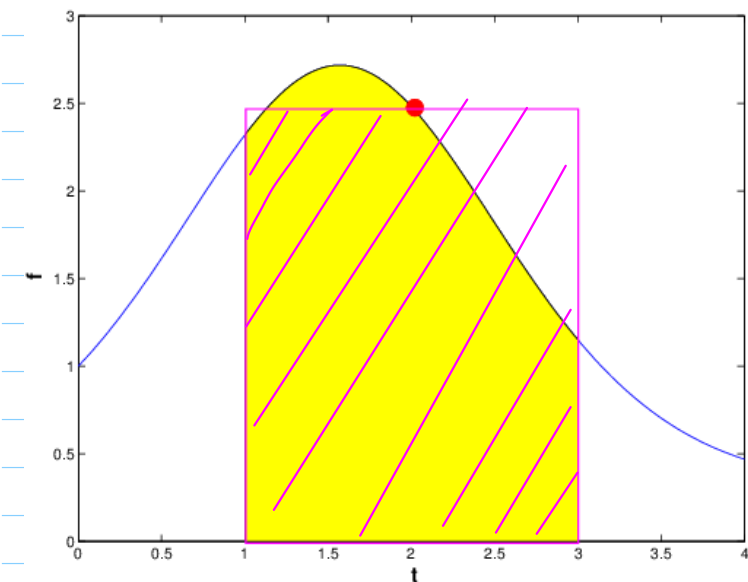
Now: $I_{\mathcal{T}} \triangleq$ Lagrange pol. interp.

$$\int_a^b f(t) dt = \sum_{j=0}^n f(t_j) \int_a^b L_j(\tau) d\tau$$

$\underbrace{\int_a^b L_j(\tau) d\tau}_{= w_j \text{ (weight)}}$
 Lagrange polynomials

② Ex. 7.3.0.3 (Midpoint rule)

$n = 1$ (degree 0), $t_z = \frac{1}{2}(b+a)$



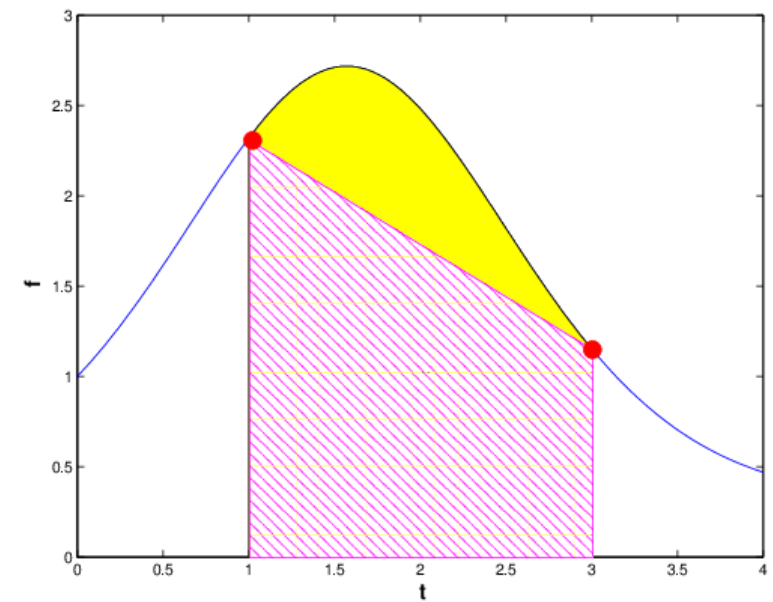
$$\int_a^b f(t) dt \approx Q_{\text{mp}}(f) = \underbrace{(b-a)}_{w_1} f\left(\frac{1}{2}(a+b)\right).$$

"midpoint"

> trapez := newtoncotes(1);

$$\hat{Q}_{\text{trp}}(f) := \frac{1}{2}(f(0) + f(1)) \quad (7.3.0.5)$$

$$\left(\int_a^b f(t) dt \approx \frac{b-a}{2} (f(a) + f(b)) \right)$$



Ex. (Trapezoidal rule)

$n = 2$ (degree 1), $t_1 = a$, $t_2 = b$

On $I = [0, 1]$: $L_1(t) = 1-t$
 $L_2(t) = t$

$\triangleright w_1 = w_2 = \frac{1}{2}$

Ex 7.3.0.4 (Newton-Cotes formulas)

Equidistant quadrature nodes in $[a, b]$

$$t_j = a + j \frac{b-a}{n-1}, \quad j = 0, \dots, n-1$$

$\triangleright$ Dangerous for $n \gg 1$: huge weights, $|w_j| \gg 1$
 with alternating sign: $\Rightarrow$ cancellation

③

Rem. 7.3.0.7 (Clenshaw-Curtis quadrature rules)

$\triangleq$ polynomial quadrature w/ Chebyshev nodes

$\triangleright$ weights $w_i > 0$

§ 7.3.0.9: Error estimates for polynomial quadrature

$$f \in C^{n+1}(I)$$

$$\|f - L_T f\|_{L^\infty(I)} \leq \frac{\|f^{(n+1)}\|_{L^\infty(I)}}{(n+1)!} \underbrace{\max_{t \in I} |(t - t_0) \cdots (t - t_n)|}_{\downarrow} \quad (6.2.2.22)$$

$$f \in C^n([a, b]) \Rightarrow \left| \int_a^b f(t) dt - Q_n(f) \right| \leq \frac{1}{n!} (b-a)^{n+1} \|f^{(n)}\|_{L^\infty([a, b])} \quad (7.3.0.10)$$

