

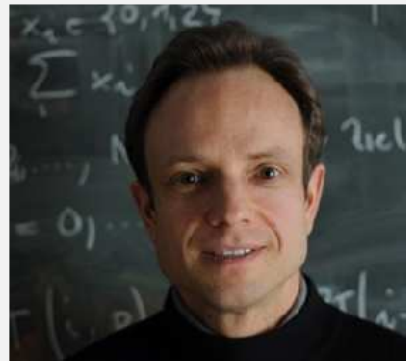
Course Video

Section 8.1: Iterative Methods for Non-Linear Systems of Equations: Introduction

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(C) Seminar für Angewandte Mathematik, ETH Zürich



Dependencies. Ex. 2.1.0.3

Duration: 10 minutes

VIII Iterative Methods for Non-Linear Systems of Equations

8.1. Introduction

Abstract NLSE : $F(x) = 0$

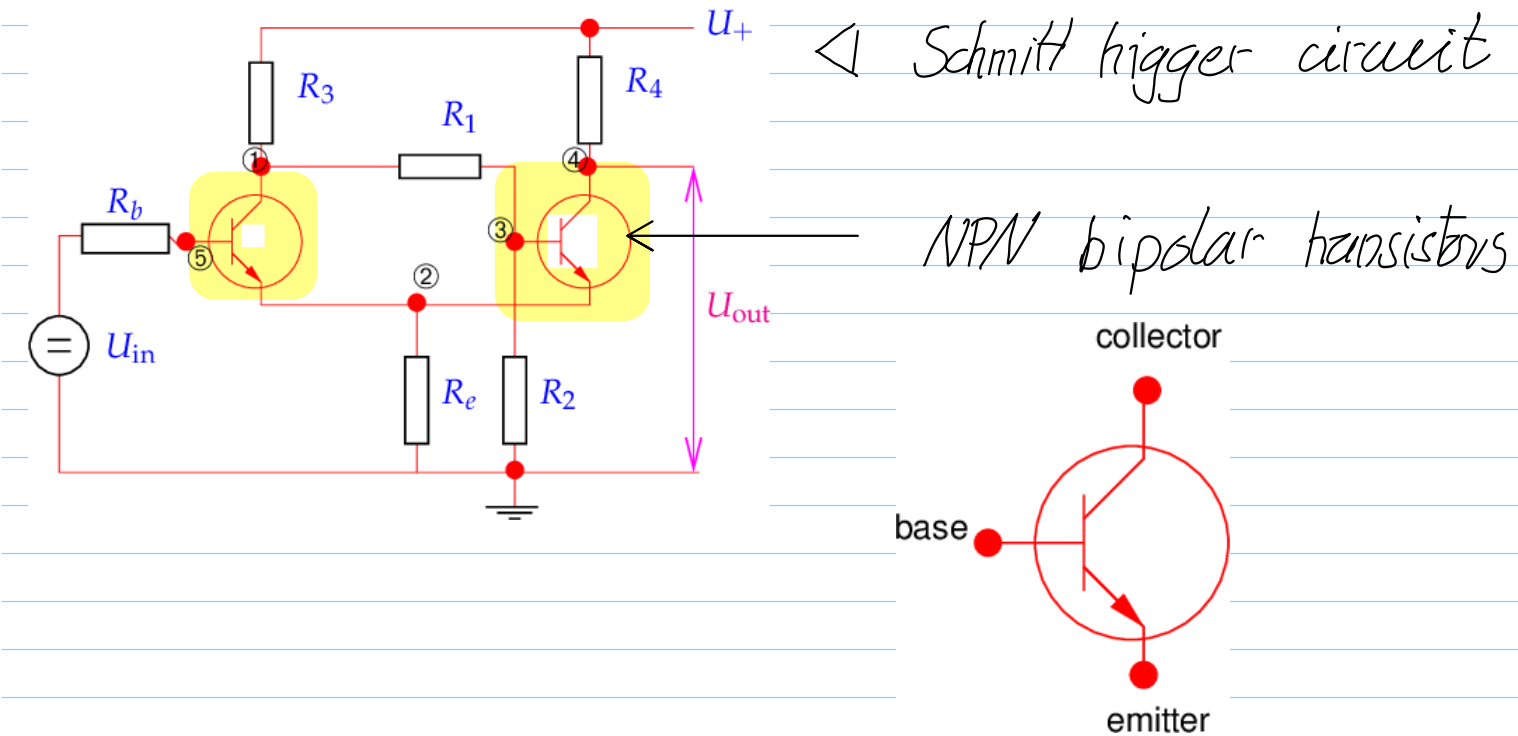
$$F : D \subset \mathbb{R}^n \longrightarrow \mathbb{R}^n$$

 $n \in \mathbb{N}$: no. of (scalar) eqns. = no. of (scalar) unknowns
LSE : $A \in \mathbb{R}^{n \times n}$, $\underline{b} \in \mathbb{R}^n$

$$Ax = \underline{b} \iff F(x) = 0, \quad F(x) := \underline{b} - Ax$$

②

Ex. 8.1.0.1. (Non-linear electric circuit)



$$I_C = I_S \left(e^{\frac{U_{BE}}{U_T}} - e^{\frac{U_{BC}}{U_T}} \right) - \frac{I_S}{\beta_R} \left(e^{\frac{U_{BC}}{U_T}} - 1 \right) = I_C(U_{BE}, U_{BC}),$$

$$I_B = \frac{I_S}{\beta_F} \left(e^{\frac{U_{BE}}{U_T}} - 1 \right) + \frac{I_S}{\beta_R} \left(e^{\frac{U_{BC}}{U_T}} - 1 \right) = I_B(U_{BE}, U_{BC}),$$

$$I_E = I_S \left(e^{\frac{U_{BE}}{U_T}} - e^{\frac{U_{BC}}{U_T}} \right) + \frac{I_S}{\beta_F} \left(e^{\frac{U_{BE}}{U_T}} - 1 \right) = I_E(U_{BE}, U_{BC}).$$

(8.1.0.2)

I_C, I_B, I_E : current in collector/base/emitter,
 U_{BE}, U_{BC} : potential drop between base-emitter, base-collector.

Nodal analysis: $U_j \triangleq$ potential at ④

$$\begin{aligned} \textcircled{1}: & R_3(U_1 - U_+) + R_1(U_1 - U_3) + I_C(U_5 - U_1, U_5 - U_2) = 0, \\ \textcircled{2}: & R_e U_2 + I_E(U_5 - U_1, U_5 - U_2) + I_E(U_3 - U_4, U_3 - U_2) = 0, \\ \textcircled{3}: & R_1(U_3 - U_1) + I_B(U_3 - U_4, U_3 - U_2) = 0, \\ \textcircled{4}: & R_4(U_4 - U_+) + I_C(U_3 - U_4, U_3 - U_2) = 0, \\ \textcircled{5}: & R_b(U_5 - U_{in}) + I_B(U_5 - U_1, U_5 - U_2) = 0. \end{aligned} \quad (8.1.0.3)$$

5 equations $\leftrightarrow$ 5 unknowns $U_1, U_2, U_3, U_4, U_5 \rightarrow \underline{x} \in \mathbb{R}^5$

$$F(\underline{x}) = 0$$

§ 8.1.0.5 (Generic/general non-linear systems of equations)

$$F(\underline{x}) = 0$$

- No general theory
- Often: F only implicit

③

▷ Review questions 8.1.0.6
[RQ 8.1.0.6.C requires knowledge about
nodal analysis of circuits; may be skipped]

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