

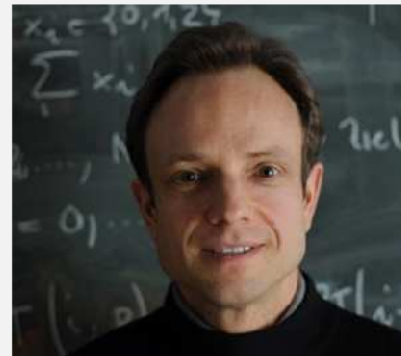
Course Video

Section 8.2.1: Iterative Methods: Fundamental Concepts

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Date: December 1, 2020

(C) Seminar für Angewandte Mathematik, ETH Zürich



Prerequisites.

- Sequences and convergence

Duration: 10 minutes

Note: Possible minor *mismatch of video and tablet notes*!

[Corrections and updates can be incorporated into tablet notes only]



VIII Iterative Methods for Non-Linear Systems of Equations

8.2. Iterative Methods

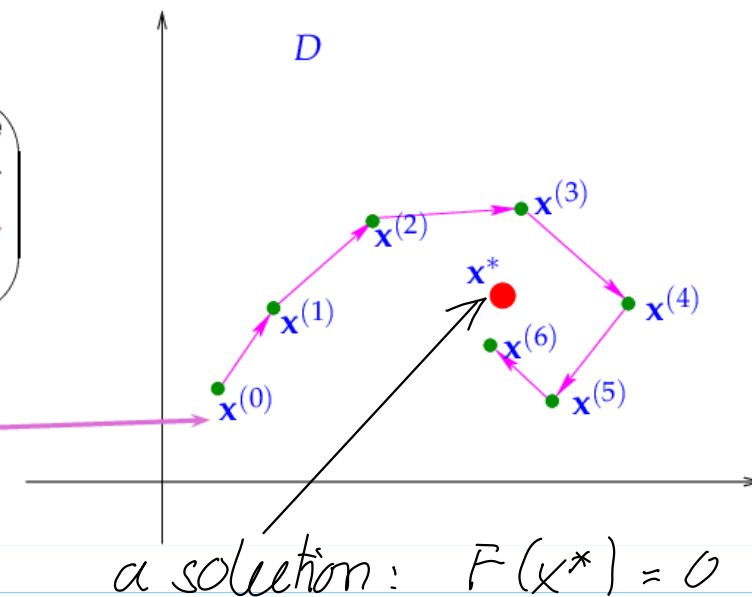
8.2.1. Fundamental Concepts

$$NLSE : F(x) = 0, \quad F: D \subset \mathbb{R}^n \rightarrow \mathbb{R}^n$$

An **iterative method** for (approximately) solving the non-linear equation $F(x) = 0$ is an algorithm generating an arbitrarily long sequence $(x^{(k)})_k$ of **approximate solutions**.

 $x^{(k)} \triangleq k\text{-th iterate}$

Initial guess



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Convergence: $\underline{x}^* := \lim_{k \rightarrow \infty} \underline{x}^{(k)}$ exists
 [Def. 8.2.1.3]

More concrete: m -point iteration

$$\underline{x}^{(k+1)} = \Phi(\underline{x}^{(k)}, \dots, \underline{x}^{(k-m+1)}), \quad k \in \mathbb{N}$$

$\hookrightarrow$ iteration function

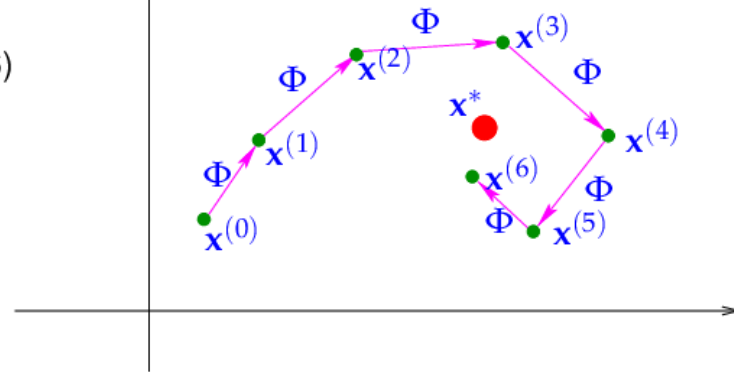
Needed: m initial guess $\underline{x}^{(0)}, \dots, \underline{x}^{(m-1)} \in \mathbb{R}^n$

Visualization of a 1-point iteration:

$$\underline{x}^{(k+1)} = \Phi(\underline{x}^{(k)}), \quad (8.2.1.6)$$

with an iteration function

$$\Phi: D \subset \mathbb{R}^n \rightarrow \mathbb{R}^n.$$



Definition 8.2.1.7. Consistency of iterative methods

A stationary m -point iterative method

$$\underline{x}^{(k+1)} = \Phi_F(\underline{x}^{(k)}, \dots, \underline{x}^{(k-m+1)}), \quad m \in \mathbb{N}, \quad (8.2.1.5)$$

is **consistent** with the non-linear system of equations $F(\underline{x}) = 0$, if and only if

$$\Phi_F(\underline{x}^*, \dots, \underline{x}^*) = \underline{x}^* \iff F(\underline{x}^*) = 0.$$

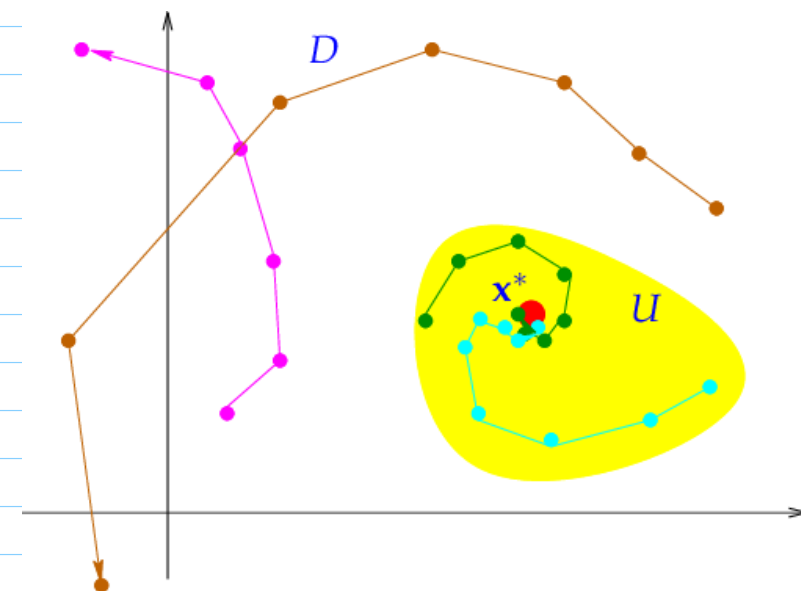
$$\left. \begin{array}{l} \underline{x}^{(k)} \text{ convt.} \\ \Phi_F \text{ consistent} \\ \Phi_F \text{ continuous} \end{array} \right\} \Rightarrow F(\underline{x}^*) = 0$$

Theorem 8.2.1.8. Consistency and convergence

The limit of a **convergent** sequence $(\underline{x}^{(k)})_{k \in \mathbb{N}_0}$ generated by an m -point stationary iterative method with a **continuous** iteration function Φ_F that is consistent with $F(\underline{x}) = 0$ is a solution:

$$\begin{aligned} \underline{x}^{(k+1)} &:= \Phi_F(\underline{x}^{(k)}, \dots, \underline{x}^{(k-m+1)}), \\ \underline{x}^* &:= \lim_{k \rightarrow \infty} \underline{x}^{(k)} \end{aligned} \Rightarrow F(\underline{x}^*) = 0.$$

§ 8.2.1.9 (Local convergence)



$\triangleleft$ Cvg. will heavily depend initial guesses

Local conv.

③

▷ Review questions 8.2.1.11

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