Numerische Mathematik, Prof. K. Nipp 5506

Musterlösung, Serie 6

1) → Matlab

2) a) $P_n = \sum_{i=0}^{n} f_i \cdot l_i(x)$ mit $l_i(x) = \prod_{j \neq i}^{n} \frac{x - x_j}{x_i - x_j}$

z.B. $l_0 = \frac{x-x_1}{x_0-x_1} \cdot \frac{x-x_2}{x_0-x_2} \cdot \frac{x-x_3}{x-x_3}$ für $n=3$

$$P_3(x) = (-2)\frac{x-1}{1}\cdot\frac{x-4}{-2}\cdot\frac{x-8}{-6} + (-1)\frac{x-1}{3}\cdot\frac{x-2}{2}\cdot\frac{x-8}{-4} + (2)\cdot\frac{x-1}{7}\cdot\frac{x-2}{6}\cdot\frac{x-4}{4}$$

$$= -\frac{1}{6}(x-1)(x-4)(x-8) + \frac{1}{24}(x-1)(x-2)(x-8) + \frac{1}{84}(x-1)(x-2)(x-4)$$

$P_3(3) = -\frac{54}{28}$

b) $P_n(x) = \frac{\sum_{i=0}^{n} \lambda_i f_i}{\sum_{i=0}^{n} \lambda_i}$ mit $\lambda_i = \frac{\lambda_i}{x - x_i}$

$\lambda_i = \frac{1}{\prod_{\substack{j \neq i \\ j>0}}^{n} (x_i - x_j)}$, z.B. $\lambda_0 = \frac{1}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$ für $n=3$

$\lambda_0 = -\frac{1}{21}$, $\lambda_1 = \frac{1}{12}$, $\lambda_2 = -\frac{1}{24}$, $\lambda_3 = \frac{1}{168}$

Für $x=3$, $\lambda_0 = -\frac{1}{42}$, $\lambda_1 = \frac{1}{12}$, $\lambda_2 = \frac{1}{24}$, $\lambda_3 = -\frac{1}{840}$ und $P_3(3) = -\frac{59}{28}$

3) → MATLAB