

Musterlösung, Serie 71) n-2 Gleichung für n Unbekannte Knoten ($n=4$):

$$b_1 f_1' + a_1 f_2' + b_2 f_3' = d_2$$

$$b_2 f_2' + a_2 f_3' + b_3 f_4' = d_3$$

"Not-a-knot" Bedingung:

$$\frac{b_1}{2} f_1' + \frac{a_1}{2} f_2' = 2c_1 + \frac{h_1}{h_1+h_2} (c_1+c_2)$$

$$\frac{a_2}{2} f_3' + b_3 f_4' = 2c_3 + \frac{h_3}{h_2+h_3} (c_2+c_3)$$

$$\begin{bmatrix} b_1 & \frac{a_1}{2} & 0 & 0 \\ b_1 & a_1 & b_2 & 0 \\ 0 & b_2 & a_2 & b_3 \\ 0 & 0 & \frac{a_2}{2} & b_3 \end{bmatrix} \begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_4' \end{bmatrix} = \begin{bmatrix} 2c_1 + \frac{h_1}{h_1+h_2} (c_1+c_2) \\ d_2 \\ d_3 \\ 2c_3 + \frac{h_3}{h_2+h_3} (c_2+c_3) \end{bmatrix}$$

$$\Rightarrow h_1 = h_2 = h_3 = 1/2$$

$$\Rightarrow b_1 = b_2 = b_3 = 1/(1/2) = 2$$

$$\Rightarrow a_1 = 2(b_1+b_2) = 8 \quad \wedge \quad a_2 = 2(b_2+b_3) = 8$$

$$\Rightarrow c_1 = \frac{f_2 - f_1}{h_1^2} = 4(1-0) = 4$$

$$\Rightarrow c_2 = \frac{f_3 - f_2}{h_2^2} = 4(0-1) = -4$$

$$\Rightarrow c_3 = \frac{f_4 - f_3}{h_3^2} = 4(-1-0) = -4$$

$$\Rightarrow d_2 = 3(c_2+c_3) = 3(4-4) = 0$$

$$\Rightarrow d_3 = 3(c_3+c_4) = 3(-4-4) = -24$$

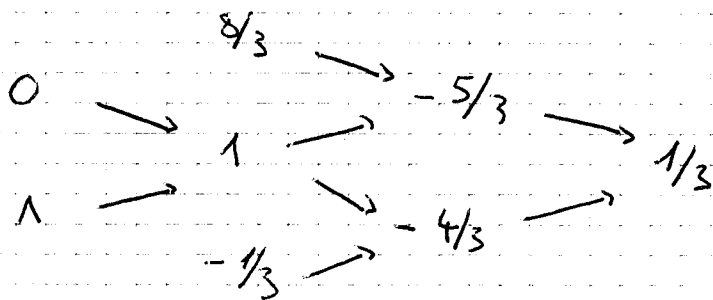
$$\Rightarrow 2c_1 + \frac{h_1}{h_1+h_2} (c_1+c_2) = 8 + \frac{1}{2}(4-4) = 8$$

$$\Rightarrow 2c_3 + \frac{h_3}{h_2+h_3} (c_2+c_3) = -8 + \frac{1}{2}(-4-4) = -12$$

$$\begin{bmatrix} 2 & 4 & & \\ 2 & 8 & 2 & \\ & 2 & 8 & 2 \\ & & 4 & 2 \end{bmatrix} \begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_4' \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ -24 \\ -12 \end{bmatrix} \Rightarrow \begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_4' \end{bmatrix} = \begin{bmatrix} 16/3 \\ -2/3 \\ -8/3 \\ -2/3 \end{bmatrix}$$

$$x = 0,25, \quad x_1 \leq x \leq x_2$$

$$t = \frac{x - x_1}{h_1} = \frac{1}{2}$$



$$Q_1(t) = 0 + (1 + (-5/3 + 1/3)(t - 1))t$$

$$Q_1(1/2) = (1 + (-5/3 + 1/3)(1/2)) \cdot 1/2$$

$$= \frac{7}{8}$$

2. $n-2$ Gleichungen für n Unbekannte (n=5)

$$b_1 f_1' + a_1 f_2' + b_2 f_3' = d_2$$

$$b_2 f_2' + a_2 f_3' + b_3 f_4' = d_3$$

$$b_3 f_3' + a_3 f_4' + b_4 f_5' = d_4$$

Für periodische Spines gilt:

$$\begin{cases} f_1 = f_5 \\ f_1' = f_5' \\ 8f_1' + 2f_2' + 2f_4' = \frac{6}{h} \end{cases}$$

$$\Rightarrow h_1 = h_2 = h_3 = h_4 = h = 1/2$$

$$\Rightarrow b_1 = b_2 = b_3 = b_4 = 2$$

$$\Rightarrow a_1 = a_2 = a_3 = 8$$

$$c_1 = \frac{f_2 - f_1}{h^2} = 4(1 - 0) = 4$$

$$c_2 = \frac{f_3 - f_2}{h^2} = 4(0 - 1) = -4$$

$$c_3 = \frac{f_4 - f_3}{h^2} = 4(-1 - 0) = -4$$

$$c_4 = \frac{f_5 - f_4}{h^2} = 4(0 - (-1)) = 4$$

$$d_2 = 3(c_1 + c_2) = 0$$

$$d_3 = 3(c_2 + c_3) = -24$$

$$d_4 = 3(c_3 + c_4) = 0$$

$$\begin{bmatrix} 8 & 2 & 0 & 0 \\ 2 & 8 & 2 & 0 \\ 0 & 2 & 8 & 2 \\ 0 & 0 & 2 & 8 \end{bmatrix} \begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_4' \end{bmatrix} = \begin{bmatrix} 0 \\ -24 \\ 0 \\ 24 \end{bmatrix} \Rightarrow \begin{bmatrix} f_1' \\ f_2' \\ f_3' \\ f_4' \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -3 \\ 0 \end{bmatrix}$$

$$x = 1/4, \quad x_1 \leq x \leq x_2, \quad t = \frac{x - x_1}{h} = \frac{1}{2}$$

$$\begin{array}{lcl} a_0 = 0 & \rightarrow & b_{-1} = 3/2 \\ a_1 = 1 & \rightarrow & b_0 = 1 \\ & & b_1 = 0 \end{array} \quad \begin{array}{lcl} & \rightarrow & c_0 = -1/2 \\ & \rightarrow & c_1 = -1 \end{array} \quad \rightarrow \quad d_0 = -1/2$$

$$Q_1(t) = 0 + \left(1 + \left(-\frac{1}{2} - \frac{1}{2}t\right)(t-1)\right)t$$

$$Q_1\left(\frac{1}{2}\right) = \left(1 + \left(-\frac{1}{2} - \frac{1}{4}\right)\left(-\frac{1}{2}\right)\right)\frac{1}{2} = 0,6875 \quad (= \frac{11}{16})$$

3. $n-2$ Gleichungen für n Unbekannte ($n=5$):

$$b_1 f_1' + a_1 f_2' + b_2 f_3' =$$

$$b_2 f_2' + a_2 f_3' + b_3 f_4' =$$

$$b_3 f_3' + a_3 f_4' + b_5 f_5' =$$

Wir wissen, dass $f_1' = f_5' = 0$.

$$\Rightarrow h_1 = h_2 = h_3 = h_4 = 1/4$$

$$\Rightarrow b_1 = b_2 = b_3 = b_4 = 4$$

$$\Rightarrow a_1 = a_2 = a_3 = 16$$

$$\Rightarrow c_1 = 16 \left(\frac{1}{4} \cdot \frac{1}{2} - (-1) \right) = 8$$

$$\Rightarrow c_2 = 16 \left(0 - (-\frac{1}{2}) \right) = 8$$

$$\Rightarrow c_3 = 16 \left(\frac{1}{2} - 0 \right) = 8$$

$$\Rightarrow c_4 = 16 \left(1 - \frac{1}{2} \right) = 8$$

$$\Rightarrow d_2 = 3(c_1 + c_2) = 48$$

$$\Rightarrow d_3 = 3(c_2 + c_3) = 48$$

$$\Rightarrow d_4 = 3(c_3 + c_4) = 48$$

$$\begin{bmatrix} 16 & 4 & 0 \\ 4 & 16 & 4 \\ 0 & 4 & 16 \end{bmatrix} \begin{bmatrix} f_2^1 \\ f_3^1 \\ f_4^1 \end{bmatrix} = \begin{bmatrix} 48 \\ 48 \\ 48 \end{bmatrix} \Rightarrow \begin{bmatrix} f_2^1 \\ f_3^1 \\ f_4^1 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 18 \\ 12 \\ 18 \end{bmatrix}$$

(~~scribble~~)

$$x = 0,1, \quad x_1 \leq x \leq x_2, \quad t = \frac{x - x_1}{h} = 0,4$$

$$\begin{array}{lcl} a_0 = -1 & \rightarrow & b_{-1} = 0 \\ & \rightarrow & b_0 = 1/2 \\ a_1 = -1/2 & \rightarrow & b_1 = \frac{18}{28} \end{array} \quad \begin{array}{l} \rightarrow d_0 = 1/2 \\ \rightarrow d_1 = 1/7 \end{array} \quad \rightarrow d_0 = -\frac{5}{14}$$

$$Q_1(t) = -1 + \left(\frac{1}{2} + \left(\frac{1}{2} + \left(-\frac{5}{14} \right) t \right) (t-1) \right) t$$

$$Q_1(0,4) = -0,8857$$

4) $\rightarrow$ Matlab