D–MATH

# Exam Quantitative Risk Management

401-3629-00L

*Last Name*

# XX

*First Name*

# XX

*Legi-Nr.*

**XX-000-000**

*Exam-No.*

***000***

*Please do not turn the page yet!*

*Please take note of the information on the answer-booklet.*

# Question 1

(a) A friend of yours tosses a fair coin and lets you choose between two bets A and B. In bet A you win 10 CHF if *head* shows up, and you loose 10 CHF if *tail* shows up. In bet B you win 30 CHF if *head* shows up, and you loose 10 CHF if *tail* shows up.

(i) [**2 Points**] Quantify the risks of the two bets in terms of Value-at-Risk at level 0.9, $\mathrm{VaR}_{0.9}$.

(ii) [**2 Points**] Quantify the risks of the two bets in terms of standard deviation, sd.

(iii) [**1 Point**] Rank the risks of the two bets in terms of a coherent risk measure $\rho$.

(b) Suppose you own a portfolio consisting of one share of stock A with current value $S_t^A = 700$ and 3 shares of stock B with current value $S_t^B = 100$ per share (both in CHF). The monthly log-returns of the stocks in % over the last 4 months are given in the following table:

| Lag $k$ | 3 | 2 | 1 | 0 |
|---|---|---|---|---|
| log-return of stock A at lag $k$ | $-2.0$ | 1.0 | $-1.0$ | 0.0 |
| log-return of stock B at lag $k$ | 1.0 | 1.5 | $-2.0$ | $-1.0$ |

(i) [**1 Point**] Express the loss $L_{t+1}$ of the portfolio over the next month as a function of the risk factor changes $X_{t+1}^A$ and $X_{t+1}^B$ given by

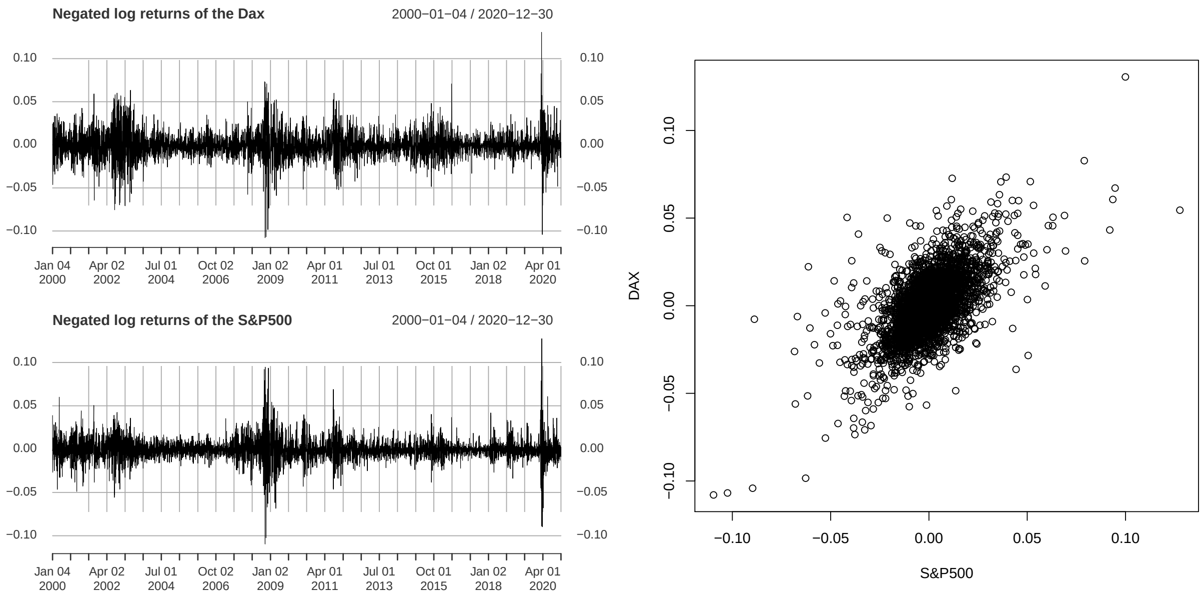
$$X_{t+1}^A = \log(S_{t+1}^A) - \log(S_t^A) \qquad \text{and} \qquad X_{t+1}^B = \log(S_{t+1}^B) - \log(S_t^B).$$

(ii) [**1 Point**] Express the *linearized* loss $L_{t+1}^\Delta$ of the portfolio as a function of $X_{t+1}^A$ and $X_{t+1}^B$.

(iii) [**4 Points**] Use historical simulation to estimate $\mathrm{VaR}_{0.6}(L_{t+1}^\Delta)$, $\mathrm{ES}_{0.6}(L_{t+1}^\Delta)$ and $\mathrm{AVaR}_{0.6}(L_{t+1}^\Delta)$.

# Question 2

(a) [**4 Points**] The following pictures show 5195 daily negative log-returns of the S&P500 and Dax from the start of 2000 to the end of 2020 (left column) along with a scatter plot of these negative log-returns (right column).




Describe four stylized facts of univariate/multivariate daily financial log-return series and relate them to the pictures.

(b) [**2 Points**] Mention a stylized fact of univariate financial log-return time series GARCH(1,1)-processes can replicate well, and explain briefly how they do so.

(c) [**4 Points**] Discuss if the following statement is true or false: "If the random variables $X_1$ and $X_2$ both follow a standard normal distribution with known correlation $\rho$, then it is possible to calculate $\mathrm{VaR}_\alpha(v_1X_1 + v_2X_2)$ for any $\alpha \in (0, 1)$ and for any $v_1, v_2 \in \mathbb{R}$."

# Question 3

Let $X$ be a random random variable with cumulative distribution function

$$F(x) = \frac{e^x}{e^x + 1}\ , \quad x \in \mathbb{R}.$$

(a) [**1 Point**] Does $X$ have a density? If no, explain why it cannot have a density. If yes, derive the density.

(b) [**2 Points**] Find all $k \in \mathbb{N} = \{1, 2, \dots\}$ such that $\mathbb{E}[|X|^k] < \infty$, providing an explanation.

(c) [**4 Points**] Does $F$ belong to the maximum domain of attraction $\mathrm{MDA}(H_\xi)$ for a standard GEV distribution $H_\xi$? If yes, what is $\xi$ and what are the normalizing sequences?

*Hint:* You may use that for any sequence $(w_n)_{n\in\mathbb{N}}$ converging to $w \in \mathbb{R}$ it holds that

$$\lim_{n\to\infty} \left(1 + \frac{w_n}{n}\right)^n = \exp(w).$$

(d) [**2 Points**] Calculate the excess distribution function $F_u(x) = \mathbb{P}[X - u \leq x \mid X > u]$, $x \geq 0$.

(e) [**4 Points**] Does there exist a parameter $\xi \in \mathbb{R}$ and a function $\beta\colon \mathbb{R} \to (0, \infty)$ such that

$$\lim_{u\to\infty} \sup_{x>0} |F_u(x) - G_{\xi,\beta(u)}(x)| = 0,$$

where $G_{\xi,\beta}$ denotes the cumulative distribution function of a GPD? If yes, for which $\xi$ and $\beta$ does this hold?

# Question 4

Let $(X_1, X_2)$ be a two-dimensional random vector with cumulative distribution function

$$F(x_1, x_2) = \begin{cases} \exp\left(-\left(x_1^{-\theta} + x_2^{-\theta}\right)^{1/\theta}\right), & \text{if } x_1 > 0 \text{ and } x_2 > 0, \\ 0, & \text{else} \end{cases}$$

for a parameter $\theta \in [1, \infty)$.

(a) **[2 Points]** Derive the two marginal cumulative distribution functions $F_1$ and $F_2$.

(b) **[2 Points]** Derive the copula of $F$.

(c) (i) **[2 Points]** Assume $\theta = 2$. Compute the probability that $X_1$ and $X_2$ both exceed their $\mathrm{VaR}_{0.95}$.

*Hint:* You may use that $\exp\left(-\left((-\log 0.95)^2 + (-\log 0.95)^2\right)^{1/2}\right) \approx 0.93$.

(ii) **[2 Points]** Show that this probability is approximately 12 times larger than the probability of the same event if $X_1$ and $X_2$ were independent.

# Question 5

(a) **[2 Points]** Describe the notions of risk and uncertainty, clearly pointing out the difference between them.

(b) **[2 Points]** Where would you place financial markets in the spectrum between risk and uncertainty? Briefly justify your answer.

(c) **[4 Points]** Let $L^2(\mathbb{P})$ be the space of all square-integrable random variables on a given probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and consider the standard deviation mapping $\mathrm{sd}\colon L^2(\mathbb{P}) \to \mathbb{R}$ given by

$$\mathrm{sd}(X) = \sqrt{\mathbb{E}[(X - \mathbb{E}[X])^2]}, \quad X \in L^2(\mathbb{P}).$$

Which properties of a coherent risk measure does sd have? Please, justify your answers.