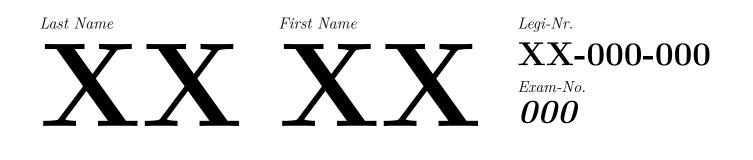


D-MATH Exam Quantitative Risk Management

401-3629-00L



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Please take note of the information on the answer-booklet.



Consider a risk measure $\rho \colon \mathcal{L} \to \mathbb{R}$ on a vector space \mathcal{L} of random variables. Show the following:

- (a) [3 Points] If ρ is positively homogeneous, then it is subadditive if and only if it is convex.
- (b) [2 Points] If ρ is subadditive, then $\rho(0) \ge 0$.
- (c) [3 Points] If ρ is subadditive and $\rho(X) \leq 0$ for all $X \in \mathcal{L}$ such that $X \leq 0$ a.s., then ρ is monotone.
- (d) [3 Points] Let $X \sim Par(\theta, \kappa)$ with cumulative distribution function

$$F(x) = 1 - \left(\frac{\kappa}{\kappa + x}\right)^{\theta}, \quad x \ge 0,$$

for parameters $\kappa > 0$ and $\theta > 1$. Calculate VaR_{α} (X) and AVaR_{α} (X).



- (a) [3 Points] Define the notion of radial symmetry of a *d*-dimensional random vector X. Show that if X is spherical, then X is radially symmetric around some $\mu \in \mathbb{R}^d$.
- (b) [4 Points] Show that two *d*-dimensional random vectors X, Y are equal in distribution if and only if $a^T X \stackrel{(d)}{=} a^T Y$ for all $a \in \mathbb{R}^d$.
- (c) [3 Points] Let a correlation matrix $C \in \mathbb{R}^{d \times d}$ be an "equicorrelation" matrix, i.e. all off-diagonal elements are equal to the same ρ . Show that $-\frac{1}{d-1} \leq \rho \leq 1$.



(a) [2 Points] Let C be the copula of (X_1, X_2, X_3) , where X_1 is a *positive* random variable with a continuous cumulative distribution function, $X_2 = 1/X_1$ and $X_3 = \exp(-X_1)$.

Explain why C is also the copula of $(X_1, -X_1, -X_1)$.

- (b) [4 Points] Derive the form of copula C from 3.(a).
- (c) [4 **Points**] Let (X, Y) be a two-dimensional random vector with a cumulative distribution function

$$F(x,y) = \frac{2x(1-e^{-2y^2})}{\sqrt{4x^2(1-e^{-2y^2})+1}}, \ x,y \ge 0.$$

Compute the copula of (X, Y).



Let X be a random variable with a cumulative distribution function

$$F(x) = \begin{cases} 1 - \frac{1}{(3x+4)^{3/4}} & \text{if } x \ge 0\\ 0 & \text{if } x < 0. \end{cases}$$

- (a) **[1 Point]** Does X have a density? If yes, can you derive it?
- (b) [1 Point] Find all $k \in \mathbb{N} = \{1, 2, ...\}$ such that $\mathbb{E}[|X|^k] < \infty$.
- (c) [3 Points] Does F belong to $MDA(H_{\xi})$ for a standard generalized extreme value distribution H_{ξ} ? If yes, what is ξ , and what are the normalizing sequences?
- (d) [2 Points] Calculate the excess distribution function $F_u(x) = \mathbb{P}[X u \le x \mid X > u], x \ge 0.$
- (e) [3 Points] Does there exist a parameter $\xi \in \mathbb{R}$ and a function β such that

$$\lim_{u \to \infty} \sup_{x > 0} |F_u(x) - G_{\xi, \beta(u)}(x)| = 0,$$

where $G_{\xi,\beta}$ denotes the cumulative distribution function of a generalized Pareto distribution? If yes, for which ξ and β does this hold?

- (a) [2 Points] Name advantages of Spearman's ρ compared to linear correlation.
- (b) [3 Points] Name advantages and disadvantages of using risk measures based on loss distributions for quantitative risk management.
- (c) [4 Points] Denote by Φ the cumulative distribution function of N(0, 1). Assuming that you can generate independent realizations of Unif(0, 1)-distributed random variables, describe an algorithm for generating realizations of a bivariate random vector $X = (X_1, X_2)$ whose distribution can be described as follows:
 - X_1 and X_2 have the same distribution.
 - The cdf F of X_1 is given by

$$F(x) = \lambda \Phi\left(\frac{x-\mu_1}{\sigma_1}\right) + (1-\lambda) \Phi\left(\frac{x-\mu_2}{\sigma_2}\right), \quad x \in \mathbb{R}$$

for some $\mu_1, \mu_2 \in \mathbb{R}$, $\sigma_1, \sigma_2 > 0$, and $\lambda \in (0, 1)$.

• The copula of X is given by a Gaussian copula $C_P \colon \mathbb{R}^2 \to [0,1]$ corresponding to a given correlation matrix $P \in \mathbb{R}^{2 \times 2}$.