> pretatione was that while a proposition is necessary when it is true, it is not therefore necessarily true simply and always. [Kneale p.214]

As we shall see later the Ockhamist system makes it possible that a proposition about the contingent future can be true now, even though its truth-value is still unknown to us. In this crucial sense Abelard's interpretation is in agreement with Prior's Ockhamist system.

Henning Boje Andersen and Jan Faye [1980] have, however, put forth a different interpretation of chapter IX. They claimed that Aristotle would probably reject the general validity of what could be called 'the law of excluded middle for statements in the future tense', i.e. for all p:

$$F(n)p \vee F(n){\sim}p$$

Given that this proposition is not valid, it must be accepted that

$$\sim F(n)p \wedge \sim F(n){\sim}p$$

may indeed be true for some proposition p. In fact, according to this interpretation the latter formula is possible for any contingent statement about the future. On the other hand, it is also clear that $F(n)p$ and $F(n){\sim}p$ cannot both be true. Therefore

$$\sim F(n)p \vee \sim F(n){\sim}p$$

must be a theorem in the Aristotelian system under this interpretation.

It is worth pointing out that this interpretation makes Aristotle's observations consistent with the aforementioned Peircean system. Thus, there is a line from the two basic interpretations of Aristotle's text presented here to Prior's two major indeterministic tense logical systems.

1.2. The Master Argument of Diodorus Cronus

Diodorus Cronus (ca. 340-280 B.C.) was a philosopher of the Megarian school [Sedley 1977]. He achieved wide fame as a logician and a formulator of philosophical paradoxes. The most well-known of these paradoxes is the so-called 'Master Argument' which in Antiquity was understood as an argument designed to prove the truth of fatalism. Unfortunately, only the premises and the conclusion of the argument are known. We know almost nothing about the way in which Diodorus used his premises in order to reach the conclusion. During the last few decades various philosophers and logicians have tried to reconstruct the argument as it might have been. The reconstruction of the Master Argument certainly constitutes a genuine problem within the history of logic. It should, however, be noted that the argument has been studied for reasons other than historical. First of all, the Master Argument has been read as an argument for determinism. Secondly, the Master Argument can be regarded as an attempt to clarify the conceptual relations between time and modality. When seen in this perspective any attempted reconstruction of the argument is important also from a systematic point of view, and this is obviously true for any version of the argument, even if it is historically incorrect. 1, 2.

Our approach in this chapter will in the first part be mainly historical. We shall comment on some of the reconstructions which have been suggested, and present an elaborated version of one of them. At the end of the chapter, we shall discuss some of the philosophical and conceptual problems related to the Master Argument.

The Master Argument is a trilemma. According to Epictetus, Diodorus argued that the following three propositions cannot all be true [Mates 1961, p.38] :

(DI) Every proposition true about the past is necessary.
(D2) An impossible proposition cannot follow from (or after) a possible one.

(D3) There is a proposition which is possible, but which neither is nor will be true.

Diodorus used this incompatibility combined with the plausibility of (D1) and (D2) to justify that (D3) is false. Assuming (D1) and (D2) he went on to define possibility and necessity as follows:

(DM) The possible is that which either is or will be true.
(DN) The necessary is that which, being true, will not be false.

In order to reconstruct the Master Argument two fundamental questions must be answered:

(1) How should 'proposition' in (Dl-3) be understood?
(2) How should 'follow' in (D2) be understood?

For the sake of completeness it should be mentioned that for some reconstructions it is also relevant whether the structure of time is assumed to be discrete or continuous.
The first of the above questions can be answered in at least two ways :

(1.1) The propositions mentioned in (D1-3) are temporally definite statements. (they are or are not the same truth value at any time)
(1.2) The Master Argument refers in fact to statements corresponding to propositional functions. (of time argument)

F.S. Michael [1976] has suggested a reconstruction of the Master Argument based on (1.1). According to Michael the truth or falsity of such statements is entirely unaffected by the time of assertion. In his version the first premise of the argument can be formulated in the following way:

(D1M) If the proposition p_0 is true at some time t' before t, then the truth of p_0 is necessary at t.
In symbols: $(T(t',p_0) \wedge t' < t) \supset N(t,p_0)$

Note that this can only be reasonable if the proposition p_0 in (D1M) itself takes the form $T(t'',r)$. Using (D1M) Michael could in fact construct an argument like the Master Argument without using (D2) directly. For his attempt at a reconstruction, however, Michael had to presuppose that a necessary proposition is true. This principle seems to be uncontroversial, but it is not implied by (D1-3) alone. His proof can be presented in the following way:

According to (D3) it is assumed that there is a proposition q_0, which is possible, but false now and also at any future time. The proposition q_0 must in the argument itself be of the form $T(t'',r)$ by Michael's assumption of (1.1). This means that the following holds:

$$M(n,q_0) \wedge T(n,\sim q_0) \wedge (\forall t\colon t>n \supset T(t,\sim q_0))$$

q_0 is possible at n $\wedge$ $\neg q_0$ is true at n $\wedge$ $\neg q_0$ is true at any point t s.t. $t>n$

Now, q_0 must be false also before n, since if for some t'

$$T(t',q_0) \wedge t'<n,$$

then (D1M) would give us $N(n,q_0)$ and therefore also $T(n,q_0)$, which would contradict the above assumption. - Hence it can be concluded that q_0 is false at any time, t, i.e.

$$t < n \supset T(t,\sim q_0)$$

for any t. It then follows from (D1M) that $N(n,\sim q_0)$. This means that $\sim M(n,q_0)$, which contradicts the above assumption about q_0 being possible at n.
Q.E.D.

It follows from the argument as reconstructed by Michael that a true proposition is necessary and a false proposition is impossible. But then it can be said that 'possible', 'true', and 'necessary' are identical qualifications of propositions. Therefore, Michael proves too much, since (DM) and (DN) are obviously meant to carry different informative content - that is, they should not be made equivalent. So there is not sufficient reason for accepting Michael's assumption regarding the status of propositions in the Master Argument. And indeed, for other and independent reasons it seems most probable that Diodorus thought of propositions as corresponding to what we today would call functions. His examples include statements like 'It is day', 'I am conversing', 'It is light'. As Mates [1961, p.36] has stated, these propositions 'are true at certain times and false at others', or equivalently, 'they become true and become false'. Furthermore, Mates could also conclude that Diodorean necessity would in most cases apply to such 'functional propositions', so generally speaking we should expect (1.2) to be the correct answer as regards the status or nature of propositions in the Master Argument. Nevertheless, Mates did not think that (D1) could make sense if 'proposition' is understood in this way [1961, p.39]. Therefore Mates' analysis apparently left us with an enigma: according to this analysis, (1.2) was the most probable answer, but Mates could not see how this assumption could be consistent with the context of the Master Argument.

However, as we shall see in the following, Prior has shown how a reading of (D1) consistent with (1.2) is in fact possible. But first we must examine the question regarding the understanding of (D2). This question can also be answered in at least two different ways:

(2.1) 'Follows' in (D2) refers to temporal order.
(2.2) 'Follows' in (D2) refers to logical implication.

Like the reconstructions of Zeller [1882] and of Copleston [1962], Rescher's reconstruction [1966] of the Master Argument is based on an assumption like (2.1), i.e. on a temporal version of (D2). Rescher assumes that the original formulation of this premise can be reformulated in the following way:

(D2x) The impossible does not follow after the possible.

(D2x) implies that what has been possible will always be possible. This 'principle of possibility-conservation' is obviously not very plausible. Even if some proposition p could once be regarded as possible, consistently with whatever else obtained at that time, some of the conditions for p may change permanently at a later time such as to make it impossible always thereafter. Moreover, Mates observed that the word used by Epictetus in (D2), which Rescher translates into 'follow after', is the same word used by Diodorus for 'is a consequent of'. It should also be noted that Chrysippus, who rejected the Master Argument, understood its second premise as referring to logical consequence rather than temporal succession [Mates 1961, p.39]. Finally, a circumstantial but important piece of evidence that (D2) is concerned with logical consequence is the fact that Diodorus studied the nature of implication very carefully. The famous debate between Diodorus and Philo of Megara precisely concerned the relation between time and implication. Their views on implication were described in the following way by Sextus Empiricus:

> according to Philo such a conditional as 'If it is day, then I am conversing' is true when it is day and I am conversing, since in that case its antecedent, 'It is day' is true and its consequent, 'I am conversing', is true; but according to Diodorus it is false, for it is possible for its antecedent, 'It is day', to be true and its consequent 'I am conversing' to be false at some time, namely, after I have become quiet... [Adv.Math. VIII, 112ff; Mates, 1961, p. 98]

This conflict between Diodorus and Philo was obviously concerned with whether one could allow the truth values of the implication to vary with time or not. As Mates [p.46] has argued, a conditional was proved to be Diodorus-true by showing that it never has a true antecedent and a false consequent. That is, Diodorus favoured what we today could call temporally strict implication, whereas Philo argued for material implication. The quotation also bears on the status of propositions, for Diodorus' argument as referred by Sextus Empiricus presupposes that propositions are understood as functions.

It appears that Diodorus regarded logical implication as very important. Therefore, it is only natural to assume that it played an important rôle in his Master Argument. We believe that (2.1) should be rejected and that (2.2) should be accepted, and A1

A2

also that it is natural to assume that the implication in question was the Diodorus-implication, which is true just in case it never has a true antecedent and a false consequent.

PRIOR'S RECONSTRUCTION

Prior's reconstruction [1967, p.32 ff.] of the Master Argument follows the line of the interpretations (1.2) and (2.2). Thus it basically adopts the same understanding of 'proposition' and consequence as we have been arguing for above. Prior uses tense- and modal operators in his reconstruction, and interprets the logical (Diodorean) consequence involved in (D2) as what is in modal logic usually called 'strict implication', symbolised by $\rightarrow$.

On these assumptions it is possible to restate the reconstruction problem. Using symbols, (D1-3) can be formulated in the following way:

(D1') $Pq \supset NPq$

(D2') $((p \rightarrow q) \wedge Mp) \supset Mq$

(D3') $(\exists r)\ (Mr \wedge {\sim}r \wedge {\sim}Fr)$

$Pq \to \Box Pq$

$((p \to q) \wedge \Diamond p) \to \Diamond q$

$\exists r (\Diamond r \wedge \neg r \wedge \neg Fr$

where F is read as 'it will be the case that...', P is read as 'it has been the case that ...'. , and $\rightarrow$ is the strict implication defined as

$$p \rightarrow q \equiv N(p \supset q)$$

We are now ready to reformulate Prior's reconstruction. In doing so, we shall at first leave aside some of the problematic points about it, in order to make the main thrust of the argument as clear as possible. We shall use the propositional function q: 'Dion is here' as an example. The reconstruction, then, runs as following way. Let us make the following two assumptions:

(P1) It is possible for Dion to be here.
In symbols: Mq

(P2) Dion is not here and he never will be here.
In symbols: ${\sim}q \wedge {\sim}Fq$

Obviously, (P1) and (P2) together make up an instance of (D3). Now intuitively speaking, if Dion is not here now and from now on never will be here, then in the 'immediate past' it was true simply that Dion never would be here. Thus, it follows from (P2) that

(P3) It has been the case that Dion never will be here.
In symbols: $P{\sim}Fq$

By substitution into (D1') we have $(P{\sim}Fq \supset NP{\sim}Fq)$. Therefore, it follows from (P3) and (D1') that

(P4) It is necessary that it has been the case that Dion never will be here. In symbols: $NP{\sim}Fq$

For the sake of exposition, it is useful to subject (P4) to two transformations. First, since N is equivalent with ${\sim}M{\sim}$, we directly obtain

(P5) It is impossible that it has not been the case that Dion never will be here. In symbols: ${\sim}M{\sim}P{\sim}Fq$

We can now make use of the common tense-logical symbol H, which is an abbreviation of ${\sim}P{\sim}$, and which may be read 'it has always been the case that ...' Using H in (P5), we get

(P6) It is impossible that it has always been the case that Dion will be here. In symbols: ${\sim}MHFq$

If Dion is here now, then at any time in the past it has been true to say 'Dion will be here'. Hence, the following implication is true:

(P7) If Dion is here, then it has always been the case that Dion will be here. In symbols: $q \rightarrow HFq$

By conjoining (P1) and (P7) we obtain $((q \rightarrow HFq) \wedge Mq)$. Using (D2') we can then deduce $MHFq$.

We have now arrived at a contradiction, since on assuming (P1) and (P2) we have derived $\sim MHFq$ (P6) as well as $MHFq$. Therefore, the combined assumption of (P1) and (P2) must be rejected.

Unfortunately, it is clear that Prior is not able to reconstruct the argument only using (D1), (D2) and (D3). In addition to these, he needs two extra premises. In order to make sure that the argument from (P2) to (P3) is valid, he must assume that

$$(\sim q \wedge \sim Fq) \supset P\sim Fq$$

or, to put it in a general form, that

(D4) $(p \wedge Gp) \supset PGp$

where $G \equiv \sim F\sim$ ('it will always be the case that...'). Furthermore, he must assume that (P7) is in fact a valid strict implication such that

(D5) $N(p \supset HFp)$

is valid in general.

Prior's proof that the three Diodorean premises (D1', D2', D3') are inconsistent given (D4) and (D5) can be summarised as a reductio ad absurdum proof in the following way:

(1)	$Mr \wedge \sim r \wedge \sim Fr$	(from D3')
(2)	Mr	(from 2)
(3)	$N(r \supset HFr)$	(from D5)
(4)	$MHFr$	(from D2, 2 & 3)
(5)	$\sim r \wedge G\sim r$	(from 1)
(6)	$PG\sim r$	(from 5 & D4)
(7)	$NPG\sim r$	(from 6 & D1)
(8)	$\sim MHFr$	(from 7; contradicts 4)

Q.E.D.

O. Becker [1960] has shown that the extra premises (D4) and (D5) can be found in the writings of Aristotle. For that reason Becker concludes that it seems reasonable to assume that the extra premises were generally accepted in antiquity.

However, Prior's addition of (D4) and (D5) is nevertheless problematic (even though the argument thus reconstructed is interesting in its own right). (D4) is in fact a rather complicated statement and not so innocuous as it may seem at first glance - observations which will indeed become clear when we are going to discuss the Ockhamist and Peircean systems. It is not very likely that Diodorus would involve such an argument without making it an explicit premise in the Master Argument. As regards (D5), we know that Diodorus used the Master Argument as a case for the definitions (DM) and (DN). That is, in the argument itself M (or N) should in a sense be regarded as primitive. It is hard to believe that Diodorus would involve a premise about N without stating it explicitly.

A NEW RECONSTRUCTION OF THE MASTER ARGUMENT

As we have argued, Mates in his excellent analysis gave all the essential information needed for a reconstruction of the Master Argument. On the basis of the considerations so far we shall suggest a very simple argument as a possible reconstruction. We shall see that the argument can be formulated without the use of complicated extra premises as it is the case in Prior's reconstruction. We shall assume that in the Master Argument certain notions regarding time and propositions are taken for granted:

(a) Time is discrete.

(b) Diodorean propositions are functions of time. Thus, propositions are functions from instants into truth values - and conversely, such functions are propositions. For the function application of a proposition p to an instant t we write $T(t,p)$.

(c) The Diodorean implication involved in (D2) can be defined in terms of present-day temporal logic as

$$(p \Rightarrow q) \text{ if and only if } (\forall t)\ (T(t,p) \supset T(t,q))$$

Ad (a): It is not possible to prove directly that Diodorus took time to be made up of temporal atoms, although there is evidence that Diodorus believed in indivisible places and bodies [Adv. Phys. II,142-143]. Richard Sorabji [p.19] has maintained that a certain passage in the works of Sextus Empiricus [M 10.86-90] indicates that Diodorus was a temporal atomist. But even if Sorabji is wrong and Diodorus was not a temporal atomist, we might still undertake a reconstruction along the lines which we have been suggesting, provided that Diodorus held something like

> (A) No proposition has a first instant of truth. If a proposition is true, it has already been true for some time.

Although we have no direct information indicating that Diodorus actually made this assumption, it is indeed very likely that he was aware of Aristotle's point of view:

> For a change can actually be completed, and there is such a thing as its end, because it is a limit. But with reference to the beginning the phrase has no meaning, for there is no beginning of a process of change, and no primary 'when' in which the change was first in progress. [Phys. 236a 12-14]

It is not unreasonable to surmise that Diodorus tried to elaborate this observation, and that this work led him to an assumption like (A). We shall, however, omit a detailed reconstruction of the master argument on the basis of (A).

Ad (b): Diodorus apparently thought of propositions as though they contained time-variables. These propositions are true at certain times and false at other times. On the other hand, Mates has maintained that "although Diodorus usually predicates

necessity of what are in effect propositional functions, it seems that in the first of his three incompatibles, necessity is predicated of a proposition" [1961, p. 39]. We shall demonstrate how an understanding of the Master Argument based on (1.2) as well as (2.2) is possible.

Ad (c): According to Mates [1961, p.45] "a conditional holds in the Diodorean sense if and only if it holds at all times in the Philonian sense". (The Philonian implication is simply the material implication). Mates has demonstrated that his conclusion is a clear consequence of a number of passages from the sources.

Note that the assumptions (a), (b), and (c) are all well documented in the known sources about Diodorus' logic. Moreover, they do not involve the modal concepts which are at stake in the argument. For these reasons (a)-(c) should not be regarded as extra premises like Prior's (D4) and (D5).

In (c), we use '$\Rightarrow$' instead of '$\rightarrow$' in order to emphasise that our definition is distinct from Prior's definition of Diodorean implication, which was

$$(p \rightarrow q) \text{ if and only if } N(p \supset q)$$

If we did not keep these two definitions apart, (c) might be seen as *defining* modality in terms of temporality. However, the Master Argument was thought to lead to such a definition, to wit, (DM) and (DN), not to presuppose it. On (c), (D2) may be rendered as

$$(p \Rightarrow q) \wedge Mp) \supset Mq$$

where the possibility-operator should be understood as a still unanalysed concept. We shall assume, however, that Diodorus accepted the usual interdefinability between necessity and possibility (as he indeed most likely did). In symbols, this means

$$M = \sim N \sim,\ N = \sim M \sim.$$

Using the assumptions (a) - (c), it is possible to reconstruct the argument.

It should be noted that although (c) defines $(p \Rightarrow q)$ in terms of temporality, it is very different from the kind of temporal definition involved in Rescher's understanding of the Diodorean 'follows'. Our understanding of $(p \Rightarrow q)$ refers to a quantification over temporal instants rather than a temporal order.

Let us assume (D3) for some statement q, e.g. 'Dion is here'. In symbols:

$$\sim q \wedge \sim Fq \wedge Mq$$

Then the statement is false now and at every future time, although Dion's being here is possible. We intend to show that the assumption of (D3) contradicts the premises (D1) and (D2).

Let r be a statement true only at the time just before the present time. Although any arbitrary statement fulfilling the requirement would do, we may choose the more intuitively appealing

r: 'The prophet says: Dion will never be here.'

From the propositional function r, we can construct the propositional function Pr, which is obviously false at any past time, true now and always in the future. We can illustrate the situation by the following figure, where the instant 'now' is represented by the number 10:

~Pr	~Pr	~Pr	Pr	Pr	Pr
~r	~r	r	~r	~r	~r
7	8	9	10	11	12
			now		
?q	?q	?q	~q	~q	~q

Clearly r is false at any instant other than 9, the instant immediately preceding the now. $\sim Pr$ is true at any past time, i.e. any instant lesser than 10. On the other hand, Pr is true now, at 10, and always thereafter. Finally, by our assumption of (D3), q is false now and always in the future. However, q might be true or false at any past time.

Since Pr is true now, we can by (D1) obtain NPr, which is equivalent with $\sim M \sim Pr$. It is also evident that

$$(q \Rightarrow \sim Pr).$$

This Diodorean implication is valid since if q is true at time t, then t must be a past time; this follows from our assumption of (D3) as illustrated in the figure. Furthermore, $\sim Pr$ is true at any past time. Therefore the antecedent can never be true when the consequent is false. But the validity of this Diodorean implication contradicts (D2), since the impossible, $\sim Pr$, follows from the possible, q. Therefore the assumption of (D3) has to be rejected.

In this way the Master Argument can be reconstructed using discrete time and the Diodorean idea of implication. We think it very likely that this was the kind of reasoning actually used by Diodorus.

It is interesting that the above argument works even if it is assumed that the first premise (D1) of the Master Argument is concerned only with propositions which are genuinely about the past. An example of a proposition which is not genuinely about the past would be 'One day ago it was the case that in two days, Dion will be here'. Such propositions should not be necessitated by (D1), although they may be necessitated on other grounds. In Prior's reconstruction, statements which are only spuriously about the past are regarded as necessary. In this way the validity of implications like $PGq \supset NPGq$ can be derived. In our reconstruction, however, such a questionable use of (D1) is completely unnecessary.

The way (D2) is used in our reconstruction bears some resemblance to one of the paradoxes of implication, since we can without loss of generality assume that q is not only false in the present and the future, but also in the past - that Dion has never been here, is not here and never will be here. In this case any proposition will follow from q in the Diodorean sense. Indeed, it is not required that there be any semantical relation between q and r in the argument. In general, if q is any proposition which is always false, then the Diodorean implication $(q \Rightarrow p)$ holds for any arbitrary proposition p; in this case, the implication obviously never has a true antecedent and a false consequent. But then we may choose any possible proposition q in order to show that p must be possible. Hence, any proposition which is always false must be possible on the assumption of (D2).

In this connection it should be noted that the ancients were aware of the paradoxes of implication. There can be no doubt that Diodorus, too, realised that any proposition which is always false, implies any other proposition.

LOGICAL DETERMINISM

It is very likely that the Master Argument was originally designed to prove fatalism or determinism. Because of the apparent plausibility of (D1) and (D2), the argument was understood as a rather strong case against (D3). The denial of (D3) is equivalent to the view that if a proposition is possible, then either it is true now or it will be true at some future time. So in a nutshell the argument is that an event which never will happen and is not happening now cannot be possible, and hence everything happening now or in the future is necessary. It should be clear, then, that the argument is interesting not only for historical reasons. Its systematical content is entirely relevant for a modern discussion of determinism, too. The present-day philosopher wanting to argue against fatalism and determinism must relate to all known versions of the Master Argument, directly or indirectly. If the fatalistic or deterministic conclusion of the Master Argument is to be avoided, at least one of the two premises (D1) and (D2) has to be denied - at any rate, that is the case as long as we accept the tacit assumption that time is a linear structure. Now for any version of the Master Argument based on that assumption we believe that it is in fact quite reasonable to deny at least one of (D1) and (D2). Let us consider the versions which have been discussed above.

As mentioned above, the second premise in Rescher's version of the Master Argument turns out to be equivalent to a 'principle of possibility-conservation'. It would certainly be reasonable to deny the validity of this principle. In Michael's version of the Master Argument the first premise, (D1M), should be denied, since it is not reasonable to view a true proposition about the future as necessary, just because it is formulated as a prophecy stated in the past. Such a proposition is about the past only in a spurious sense. Regarding (D1) in Prior's reconstruction we can make a similar observation. The statement

'It has been that Dion never will be here', (in symbols: $P{\sim}Fq$)

should not be counted as necessary even if it is true. Even if we accept ${\sim}q$, ${\sim}Fq$, and $P{\sim}Fq$, there is no a priori reason to exclude the conceptual possibility of Dion's being here at some future time, or his 'having always been going to be here', i.e. MFq and $MPGq$. Therefore, the way in which (D1) is used in Prior's version of the argument should certainly be questioned.

In our reconstruction, we do not have to assume any more than the necessity of propositions which are genuinely about the past. When (D1) is seen in this way, it appears reasonable, whereas (D2) should be rejected if time is linear. The reason is that if there is a propositional function q which is possible but never true, then our version of (D2) implies that any absurdity $(p \wedge {\sim}p)$ also becomes possible. Obviously, it is not acceptable to regard an absurdity as being possible. Given that time is linear it seems entirely reasonable to deny (D2).

Prior himself questioned the validity of (D5) i.e.

(D5) $N(p \supset HFp)$

If we understand 'will be' as 'determinately will be', then (D5) can certainly be denied, as in fact it is in the Peircean system, which Prior elaborated and to which he indeed preferred himself. We shall return to this system in part 2.

SOME CONCEPTUAL CONSIDERATIONS

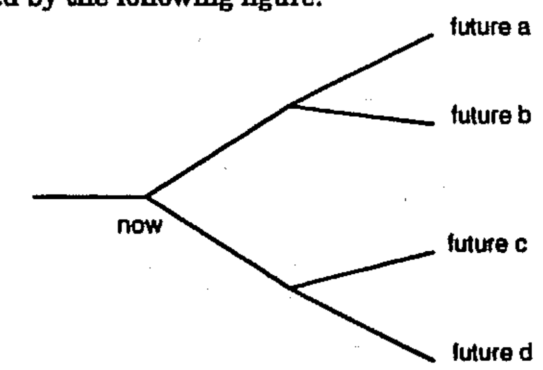
The Master Argument can also be read as an attempt to relate the modal concepts of possibility and necessity to the concept of time. The various versions of the argument emphasise the impact of temporal indices on the operators of possibility and necessity. For instance, what is possible now need not be possible in the future. And what is now not necessary but a mere possibility, can become necessary in the future. It is obvious that the notion of modality involved in such assumptions should be linked to the idea of time. A proposition is necessary if it is 'now-unpreventable', and a proposition is possible if its negation is 'now-preventable'. In formulating his argument Diodorus was aiming at a justification of his definitions of possibility and necessity, (DM) and (DN), which were:

(DM) The possible is that which either is or will be true.
(DN) The necessary is that which, being true, will not be false.

But if these definitions are accepted, and if time is understood as a linear structure, then we are led to some kind of fatalism or determinism.

As we have seen, we do not have to accept (DM) and (DN) on account of the argument itself, since at least one of the premises (D1) and (D2) should be rejected if time is implicitly or explicitly understood to be a linear structure. However, the picture is somewhat different if we avail ourselves of the modern notion of branching time: that is, if time is considered to be a branching structure, it is not representable as a subset of the real numbers, and both (D1) and (D2) as understood in our reconstruction become plausible. In part 2 we shall examine the notion of

branching time in detail. The basic idea can, however, easily be illustrated by the following figure:



The central idea is that for any given 'now' there are a number of possible and different futures - sometimes called the 'forking paths into the future'. Just one of these will become actualised in the course of time. In this kind of structure a propositional function cannot be represented by a series of truth-values. Rather, it must be represented as a complex structure of values. It should not be too hard to see that if the complex structures of branching time are discrete, then our new version of the Master Argument is still valid. The premises (D1) and (D2) as understood in our version can be accepted within all theories of branching time, in which case the conclusion of the Master Argument also has to be accepted within these theories. An adequate conception of the notion of 'possibility' can then be captured by the formula

$$Mr \equiv (r \vee Fr)$$

Obviously this means that the definitions (DM) and (DN) should also be adopted in theories of branching time. In fact, the very use of the idea of 'possible futures' can be understood as an acceptance of the conclusion of the Master Argument, since it is evident that if time is branching then any possibility must belong to some possible future. So when we investigate the Master Argument from the perspective of the historical development of the logical study of time, the argument turns out to be a demonstration of a fundamental relationship between time and modality rather than a case for fatalism or determinism.

The relation between time and modality and the attempt to define modality in terms of tense were very important to the founder of modern symbolic tense logic, A. N. Prior. As we shall see in part 2, Prior elaborated the formula above into a very complex and conceptually refined definition - his so-called fourth grade of tense-logical involvement, wherein the concept of modality becomes entirely absorbed by this tense logic. This fourth grade expressed Prior's own conception of time.

1.3. THE STUDY OF TENSES IN THE MIDDLE AGES

The Diodorean Master Argument can be seen as an example of that interest in the logic of statements involving time which is part of a tradition dating back to Aristotle and other Ancient philosophers. The Scholastic logicians in particular made a number of original contributions to tense-logic. We shall now devote a few chapters to a brief survey of the most important of these contributions.

Medieval logicians were engaged in an attempt to develop a logic of natural language. With this objective they had to take heed of the fact that some statements do not have fixed truth-values. A proposition like 'Socrates is alive' is true when Socrates is alive, and it is false when he is not alive. Therefore it is an integral part of medieval logic that the truth-value of a proposition can vary from time to time. For the same reasons it was natural, indeed inevitable, for them to analyse tensed statements in their logical studies. It was an important goal of theirs to be able to describe the logical content of propositions about past and future events.

The Scholastic logicians discussed the status of tensed statements with a view to theological problems. In the course of time the difference between statements such as 'Christ was born', 'Christ is born', and 'Christ will be born' had given rise to a theological and logical problem. On the one hand, a distinction between the three forms from a purely logical point of view was considered legitimate. On the other hand, some claimed that there was in principle no difference between what had been believed by the prophets (the third form), the contemporaries of Jesus (the second form), and what has been believed by Christians in all the succeeding centuries (the first form). The object of the faith is therefore the same one. But how can the unity of faith and its independence of time be maintained, when its main tenets are described by statements whose meanings seem to vary in time in the same manner as other tensed statements?

There were two different solutions in the Middle Ages, as pointed out by Nuchelman [1980, p.133]. Firstly, there was a

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