Tautologies

In this section we give a list of some of the most important tautologies. Many of them have been used explicitly and implicitly in several formal proofs.

(A.1) $\vdash \varphi \to \varphi$
(A.0) $\vdash \varphi \leftrightarrow \varphi$

(B) $\{\psi, \varphi\} \vdash \varphi \wedge \psi$

(C) $\vdash (\psi \to \varphi) \to (\psi \to \forall \nu \varphi)$ [for $\nu \notin \mathrm{free}(\psi)$]

(D.1) $\{\varphi_0 \to \varphi_1, \varphi_1 \to \varphi_2\} \vdash \varphi_0 \to \varphi_2$
(D.2) $\{\varphi_0 \to \psi, \varphi_1 \to \psi\} \vdash (\varphi_0 \vee \varphi_1) \to \psi$
(D.3) $\{\psi \to \varphi_0, \psi \to \varphi_1\} \vdash \psi \to (\varphi_0 \wedge \varphi_1)$

(E) $\vdash \varphi \to \big(\psi \to (\varphi \wedge \psi)\big)$

(F.1) $\vdash \varphi \to \neg\neg\varphi$
(F.2) $\vdash \neg\neg\varphi \to \varphi$
(F.0) $\vdash \varphi \leftrightarrow \neg\neg\varphi$

(G.1) $\vdash (\varphi \to \psi) \to (\neg\psi \to \neg\varphi)$
(G.2) $\vdash (\neg\psi \to \neg\varphi) \to (\varphi \to \psi)$
(G.0) $\vdash (\varphi \to \psi) \leftrightarrow (\neg\psi \to \neg\varphi)$

(H.0) $\{\varphi \leftrightarrow \psi\} \vdash \neg\varphi \leftrightarrow \neg\psi$
(H.1) $\{\varphi \leftrightarrow \varphi', \psi \leftrightarrow \psi'\} \vdash (\varphi \to \psi) \leftrightarrow (\varphi' \to \psi')$
(H.2) $\{\varphi \leftrightarrow \varphi', \psi \leftrightarrow \psi'\} \vdash (\varphi \vee \psi) \leftrightarrow (\varphi' \vee \psi')$
(H.3) $\{\varphi \leftrightarrow \varphi', \psi \leftrightarrow \psi'\} \vdash (\varphi \wedge \psi) \leftrightarrow (\varphi' \wedge \psi')$

(I.1) $\vdash (\varphi_1 \wedge \varphi_2) \leftrightarrow (\varphi_2 \wedge \varphi_1)$
(I.2) $\vdash (\varphi_1 \wedge \varphi_2) \wedge \varphi_3 \leftrightarrow \varphi_1 \wedge (\varphi_2 \wedge \varphi_3)$

(J.1) $\vdash (\varphi_1 \vee \varphi_2) \leftrightarrow (\varphi_2 \vee \varphi_1)$
(J.2) $\vdash (\varphi_1 \vee \varphi_2) \vee \varphi_3 \leftrightarrow \varphi_1 \vee (\varphi_2 \vee \varphi_3)$

(K.1) $\vdash (\neg\varphi \vee \psi) \rightarrow (\varphi \rightarrow \psi)$
(K.2) $\vdash (\varphi \rightarrow \psi) \rightarrow (\neg\varphi \vee \psi)$
(K.0) $\vdash (\varphi \rightarrow \psi) \leftrightarrow (\neg\varphi \vee \psi)$

(L.1) $\vdash (\neg\varphi \vee \neg\psi) \rightarrow \neg(\varphi \wedge \psi)$
(L.2) $\vdash \neg(\varphi \wedge \psi) \rightarrow (\neg\varphi \vee \neg\psi)$
(L.0) $\vdash \neg(\varphi \wedge \psi) \leftrightarrow (\neg\varphi \vee \neg\psi)$

(M.1) $\vdash \big(\varphi_1 \rightarrow (\varphi_2 \rightarrow \varphi_3)\big) \leftrightarrow \big((\varphi_1 \wedge \varphi_2) \rightarrow \varphi_3\big)$
(M.2) $\vdash \neg(\varphi \vee \psi) \leftrightarrow (\neg\varphi \wedge \neg\psi)$

(N.1) $\vdash (\varphi_1 \wedge \varphi_2) \vee \varphi_3 \rightarrow (\varphi_1 \vee \varphi_3) \wedge (\varphi_2 \vee \varphi_3)$
(N.2) $\vdash (\varphi_1 \vee \varphi_3) \wedge (\varphi_2 \vee \varphi_3) \rightarrow (\varphi_1 \wedge \varphi_2) \vee \varphi_3$
(N.0) $\vdash (\varphi_1 \wedge \varphi_2) \vee \varphi_3 \leftrightarrow (\varphi_1 \vee \varphi_3) \wedge (\varphi_2 \vee \varphi_3)$

(O) $\vdash (\varphi_1 \vee \varphi_2) \wedge \varphi_3 \leftrightarrow (\varphi_1 \wedge \varphi_3) \vee (\varphi_2 \wedge \varphi_3)$

(P.1) $\vdash x = y \leftrightarrow y = x$
(P.2) $\vdash (x = y \wedge y = z) \rightarrow x = z$

(Q.1) $\vdash \varphi(\nu) \leftrightarrow \varphi(\nu')$ [if ν' does not appear in $\varphi(\nu)$]
(Q.2) $\vdash \exists\nu\varphi(\nu) \leftrightarrow \exists\nu'\varphi(\nu')$ [if ν' does not appear in $\varphi(x)$]
(Q.3) $\vdash \forall\nu\varphi(\nu) \leftrightarrow \forall y\varphi(\nu')$ [if ν' does not appear in $\varphi(\nu)$]

(R.1) $\{\varphi \leftrightarrow \psi\} \vdash \forall\nu\varphi \leftrightarrow \forall\nu\psi$
(R.2) $\{\varphi \leftrightarrow \psi\} \vdash \exists\nu\varphi \leftrightarrow \exists\nu\psi$

(S.1) $\vdash \neg\exists\nu\varphi \rightarrow \forall\nu\neg\varphi$
(S.2) $\vdash \neg\forall\nu\neg\varphi \rightarrow \exists\nu\varphi$
(S.3) $\vdash \exists\nu\varphi \rightarrow \neg\forall\nu\neg\varphi$
(S.0) $\vdash \exists\nu\varphi \leftrightarrow \neg\forall\nu\neg\varphi$

(T) $\vdash \forall\nu\varphi \leftrightarrow \neg\exists\nu\neg\varphi$

(U.1) $\vdash \exists x \exists y \varphi \leftrightarrow \exists y \exists x \varphi$
(U.2) $\vdash \exists x \exists x \varphi \leftrightarrow \exists x \varphi$
(U.3) $\vdash \forall x \exists x \varphi \leftrightarrow \exists x \varphi$
(U.4) $\vdash \exists x \forall x \varphi \leftrightarrow \forall x \varphi$

(V.1) $\vdash \big(\exists x \varphi \wedge \exists y \psi\big) \leftrightarrow \big(\exists x \exists y (\varphi \wedge \psi)\big)$ [for $x \notin \mathrm{free}(\psi)$, $y \notin \mathrm{free}(\varphi)$]
(V.2) $\vdash \big(\forall x \varphi \wedge \forall y \psi\big) \leftrightarrow \big(\forall x \forall y (\varphi \wedge \psi)\big)$ [for $x \notin \mathrm{free}(\psi)$, $y \notin \mathrm{free}(\varphi)$]
(V.3) $\vdash \big(\exists x \varphi \wedge \forall y \psi\big) \leftrightarrow \big(\exists x \forall y (\varphi \wedge \psi)\big)$ [for $x \notin \mathrm{free}(\psi)$, $y \notin \mathrm{free}(\varphi)$]
(V.4) $\vdash \big(\exists x \varphi \wedge \psi\big) \leftrightarrow \big(\exists x (\varphi \wedge \psi)\big)$ [for $x \notin \mathrm{free}(\psi)$]
(V.5) $\vdash \big(\forall x \varphi \wedge \psi\big) \leftrightarrow \big(\forall x (\varphi \wedge \psi)\big)$ [for $x \notin \mathrm{free}(\psi)$]

(W.1) $\vdash \big(\exists x \varphi \vee \exists y \psi\big) \leftrightarrow \big(\exists x \exists y (\varphi \vee \psi)\big)$ [for $x \notin \mathrm{free}(\psi)$, $y \notin \mathrm{free}(\varphi)$]
(W.2) $\vdash \big(\forall x \varphi \vee \forall y \psi\big) \leftrightarrow \big(\forall x \forall y (\varphi \vee \psi)\big)$ [for $x \notin \mathrm{free}(\psi)$, $y \notin \mathrm{free}(\varphi)$]
(W.3) $\vdash \big(\exists x \varphi \vee \forall y \psi\big) \leftrightarrow \big(\exists x \forall y (\varphi \vee \psi)\big)$ [for $x \notin \mathrm{free}(\psi)$, $y \notin \mathrm{free}(\varphi)$]
(W.4) $\vdash \big(\exists x \varphi \vee \psi\big) \leftrightarrow \big(\exists x (\varphi \vee \psi)\big)$ [for $x \notin \mathrm{free}(\psi)$]
(W.5) $\vdash \big(\forall x \varphi \vee \psi\big) \leftrightarrow \big(\forall x (\varphi \vee \psi)\big)$ [for $x \notin \mathrm{free}(\psi)$]