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Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 6

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26. (a) What is the order of the group $(\mathbb{Z}_{17}^*, \cdot)$?
    (b) Compute the order of 2 in the group $(\mathbb{Z}_{17}^*, \cdot)$.
    (c) Compute the order of 6 in the group $(\mathbb{Z}_{17}^*, \cdot)$.
    (d) Find an element in $\mathbb{Z}_{17}^*$ of order 4.
    (e) Which of the numbers $2^{888} - 1$, $6^{888} - 1$ and $15^{444} + 1$ is a multiple of 17 ?
27. (a) Why is $(\mathbb{Z}_{15}^*, \cdot)$ not a group?
    (b) For which $a \in \mathbb{Z}_{15}^*$ there is a number $b \in \mathbb{Z}_{15}^*$ such that $a \cdot b \equiv_{15} 1$?
28. Let $m \geq 2$. An element $a \in \mathbb{Z}_m^*$ is called *invertible* if there is a number $b \in \mathbb{Z}_m^*$ such that $a \cdot b \equiv_m 1$.
    Show that $a \in \mathbb{Z}_m^*$ is invertible if and only if $\gcd(a, m) = 1$.
29. Let $m \geq 2$ and let $\varphi(m) = \{a \in \mathbb{Z}_m^* : \gcd(a, m) = 1\}$.
    Show that $(\varphi(m), \cdot)$ is a group. (Notice that if p is prime, then $\varphi(p) = \mathbb{Z}_p^*$.)
30. (a) What is the order of the group $(\varphi(18), \cdot)$?
    (b) What is the order of 7 in $(\varphi(18), \cdot)$?
    (c) What is the order of 5 in $(\varphi(18), \cdot)$?
    (d) Which of the numbers $5^{333} - 1$, $43^{333} - 1$ and $11^{333} + 1$ is a multiple of 18 ?