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Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 8

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36. Let n be a positive integer. Show that

$$\begin{array}{rrcl} \varphi: & (\mathrm{GL}(n), \cdot\,) & \to & (\mathbb{R}^*, \cdot\,) \\ & A & \mapsto & \det(A) \end{array}$$

is a surjective homomorphism and calculate its kernel.
Can φ be an isomorphism?

37. Let $\mathbb{U} = \big\{ z \in \mathbb{C} : |z| = 1 \big\}$.
Find a surjective homomorphism from $(\mathbb{C}^*, \cdot\,)$ to $(\mathbb{U}, \cdot\,)$ and calculate its kernel.

38. Let G be a group and let $x \in G$. Show that

$$\begin{array}{rrcl} \varphi_x: & G & \to & G \\ & a & \mapsto & xax^{-1} \end{array}$$

is an automorphism.

39. Let $m \in \mathbb{Z}$. Show that

$$\begin{array}{rrcl} \varphi: & (\mathbb{Z}, +) & \to & (\mathbb{Z}, +) \\ & x & \mapsto & mx \end{array}$$

is an endomorphism and calculate its kernel.
Can φ be an automorphism?

40. Find 7 homomorphisms from $(\mathbb{Z}, +)$ to $(\mathbb{Z}_{12}, +)$ and compute their kernels as well as their images.