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Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 10, 2002

You may download the ps-file from
`http://www.math.berkeley.edu/~halbeis/4students/gt.html`

Please hand in your solutions (stapled together with your full name on the first page) at the lecture on Monday, 29 April 2002.

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46. Let n and m be a positive integers such that m divides n.
    (a) Show that $n\mathbb{Z} \trianglelefteq m\mathbb{Z}$, $n\mathbb{Z} \trianglelefteq \mathbb{Z}$ and $m\mathbb{Z} \trianglelefteq \mathbb{Z}$.
    (b) Show that $\mathbb{Z}/m\mathbb{Z}$ is isomorphic to $(\mathbb{Z}/n\mathbb{Z})/(m\mathbb{Z}/n\mathbb{Z})$

47. Let $\pi \in S_7$ be given by

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 6 & 4 & 7 & 5 & 3 \end{pmatrix}.$$

    (a) Write the permutation π in terms of disjoint cycles.
    (b) Write π^{-1}, π^{14} and π^{-27} in terms of disjoint cycles.

48. Let $\pi \in S_7$ as in question 47.
    (a) Write π in terms of transpositions.
    (b) Write π in terms of transpositions of the form $(1, j)$, where $2 \leq j \leq 7$.

49. Show that the transpositions $(1, 2), (2, 3), \ldots, (n - 1, n)$ generate S_n.

50. Show that the two permutations $(1, 2)$ and $(1, 2, \ldots, n)$ generate S_n.

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Office hours: Monday 3 pm–4 pm, Friday 1 pm–2 pm