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Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 11, 2002

You may download the ps-file from
http://www.math.berkeley.edu/~halbeis/4students/gt.html

You don't have to hand in your solutions.

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51. Let $\sigma \in S_8$ be given by

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 4 & 6 & 7 & 1 & 3 & 2 & 8 \end{pmatrix} .$$

(a) Write σ^{-1} and σ^{-333} in terms of disjoint cycles.

(b) Does σ belong to A_8 ?

(c) Does $(1,3,5,7)\sigma(2,4,8)\sigma^2$ belong to A_8 ?

52. Show that for any $n \geq 3$, A_n is generated by the set of all cycles of length 3.

53. (a) Show that if a group G has order pq^2, where p and q are two different prime numbers such that $p \nmid q^2 - 1$ or $q \nmid p - 1$, then G cannot be simple.

(b) Show that a group of order 605 cannot be simple.

54. Show that a group of order 45 is always abelian.

Hint: Use that a group of order p^2, where p is prime, is always abelian.

55. (a) How many subgroups of order 2, 7 and 9 respectively a group of order 126 could have?

(b) Can a group of order 126 be simple?

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