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Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 2, 2002

The ps-file you may download from
http://www.math.berkeley.edu/~halbeis/4students/gt.html

Please hand in your solutions (stapled together with your full name on the first page) at the lecture on Monday, 11 February 2002.

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6. For $m \in \mathbb{Z}$, let $m\mathbb{Z} := \{mx : x \in \mathbb{Z}\}$.
   (a) Describe the sets $0\mathbb{Z}$, $17\mathbb{Z}$, $(-5)\mathbb{Z}$, $5\mathbb{Z}$.
   (b) Show that for any $m \in \mathbb{Z}$, $m\mathbb{Z} = (-m)\mathbb{Z}$.
   (c) Show that for any $m \in \mathbb{Z}$, $(m\mathbb{Z}, +)$ is an abelian group.

7. On the set $(\mathbb{Q} \times \mathbb{Q})^* := (\mathbb{Q} \times \mathbb{Q}) \setminus \{(0,0)\}$ we define the binary operation "$\bullet$" as follows:
   $$(p_1, q_1)\bullet(p_2, q_2) := (p_1p_2 - q_1q_2,\ p_1q_2 + q_1p_2)$$
   Show that $\big((\mathbb{Q} \times \mathbb{Q})^*, \bullet\big)$ is an abelian group.

8. Use the templates to make the five Platonic solids. The templates you may also download from
   http://www.bath.ac.uk/~masgcs/book2/

9. Let T, C, O, D and I donote the groups of rigid motions of the five Platonic solids, called *Tetrahedron, Cube, Octahedron, Dodecahedron* and *Icosahedron*.
   Compute $|T|$, $|C|$, $|O|$, $|D|$ and $|I|$.

10. (a) Show that the group T is not commutative.
    (b) Find a non-neutral element $\sigma \in T$ with σ^2 is the identity.
    (c) Find a non-neutral element $\tau \in T$ with τ^3 is the identity.
    Hint: If `A,B,C,D` denote the faces of the tetrahedron, then each element of T corresponds to a permutation of the set $\{$`A,B,C,D`$\}$.

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Office hours: Monday 3 pm–4 pm, Friday 1 pm–2 pm