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Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 4, 2002

You may download the ps-file from
`http://www.math.berkeley.edu/~halbeis/4students/gt.html`

Please hand in your solutions (stapled together with your full name on the first page) at the lecture on Monday, 25 February 2002.

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16. Let G be a group with neutral element e in which for all $x \in G$ we have $xx = e$. Show that G is abelian.

17. Let $H_2 = \langle 2 \rangle$, $H_3 = \langle 3 \rangle$ and $H_{27} = \langle 27 \rangle$ be the subgroups of $(\mathbb{Q}^*, \cdot)$, generated by 2, 3 and 27, respectively.

    (a) Describe the elements of the subgroups H_2, H_3 and H_{27}.

    (b) Show that $H_2 \cap H_3 = \langle 1 \rangle$.

    (c) Show that $H_3 \cap \langle -\frac{1}{3} \rangle = \langle 9 \rangle$.

    (d) Show that $H_{27} < H_3$.

    (e) Evaluate $|H_3 : H_{27}|$.

18. Let $\mathbb{R}^{>0}$ be the group of positive real numbers together with multiplication and let $\mathbb{R}^*$ be the group of non-zero real numbers together with multiplication.

    (a) Show that $\mathbb{R}^{>0}$ is a subgroup of $\mathbb{R}^*$.

    (b) Evaluate $|\mathbb{R}^* : \mathbb{R}^{>0}|$.

    (c) Find a transversal for $\mathbb{R}^{>0}$ in $\mathbb{R}^*$ which is also a subgroup of $\mathbb{R}^*$.

19. Let $\mathbb{Z}$ be the group of integers together with addition.

    (a) Evaluate $|\mathbb{Z} : 7\mathbb{Z}|$.

    (b) Find a transversal for $7\mathbb{Z}$ in $\mathbb{Z}$ which contains only even numbers.

20. Let T be the tetrahedron-group. Let δ be the rotation through $2\pi/3$ about an axis joining a vertex to the centre of the opposite face and let ρ be the rotation through π about an axis joining midpoints of opposite edges.

    (a) Evaluate the order of δ and ρ, respectively.

    (b) Evaluate the order of the subgroup $\langle \delta \rangle \cap \langle \rho \rangle$.

    (c) Show that $\langle \{\delta, \rho\} \rangle = T$.

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Office hours: Monday 3 pm–4 pm, Friday 1 pm–2 pm