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Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 5, 2002

You may download the ps-file from `http://www.math.berkeley.edu/~halbeis/4students/gt.html`

Please hand in your solutions (stapled together with your full name on the first page) at the lecture on Monday, 4 March 2002.

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21. Let $(G, \circ)$ be a finite group of order n and let $x \in G$.

    (a) Show that if the order of x is k, then $k \mid n$ (in words, k divides n).

    (b) Show that $\underbrace{x \circ x \circ \ldots \circ x}_{n\text{-times}} = e$, where e is the neutral element of G.

22. (a) Show that $C_3 \times C_4$ is a cyclic group of order 12. Hence, $C_3 \times C_4$ is essentially the same group as C_{12}.

    (b) Show that $C_2 \times C_4$ is not a cyclic group.

    (c) Let $C_2 \times C_2 = \{e, a, b, c\}$ where e is the neutral element. Give the Cayley table for $C_2 \times C_2$ and evaluate the orders of a, b and c respectively.

23. Let $\mathbb{C}^*$ be the abelian group of non-zero complex numbers under multiplication. Let $\mathbb{U} = \{e^{i\varphi} : \varphi \in \mathbb{R}\}$ and let $\mathbb{R}^{>0}$ be the set of positive real numbers.

    (a) Show that $\mathbb{R}^{>0}$ is a subgroup of $\mathbb{C}^*$.

    (b) Show that $\mathbb{U}$ is a subgroup of $\mathbb{C}^*$.

    (c) Plot some of the sets $\mathbb{C}^*/\mathbb{U}$ and $\mathbb{C}^*/\mathbb{R}^{>0}$ on an Argand diagram.

    (d) Find three disjoint transversals for $\mathbb{R}^{>0}$ in $\mathbb{C}^*$.

    (e) Find three disjoint transversals for $\mathbb{U}$ in $\mathbb{C}^*$.

24. Let $O(n)$ be the group of all orthogonal $n \times n$ matrices under multiplication and let $SO(n) < O(n)$ be the group of all orthogonal $n \times n$ matrices with determinant 1.

    (a) Evaluate $|O(n) : SO(n)|$.

    (b) Find a transversal for $SO(4)$ in $O(4)$.

25. Let T be the tetrahedron-group and let ρ_1, ρ_2 and ρ_3 be the three rotations through π about the axes joining midpoints of opposite edges.

    (a) Show that $\langle\{\rho_1, \rho_2, \rho_3\}\rangle$ is a subgroup of T of order 4.

    (b) Give the Cayley table for $\langle\{\rho_1, \rho_2, \rho_3\}\rangle$.

    (c) Show that $\langle\{\rho_1, \rho_2, \rho_3\}\rangle$ is essentially the same group as $C_2 \times C_2$.

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Office hours: Monday 3 pm–4 pm, Friday 1 pm–2 pm