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Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 7, 2002

You may download the ps-file from
`http://www.math.berkeley.edu/~halbeis/4students/gt.html`

Please hand in your solutions (stapled together with your full name on the first page) at the lecture on Wednesday, 20 March 2002.

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31. Let G be a group. Show that if $H, K \trianglelefteq G$, then $(H \cap K) \trianglelefteq G$.

32. Let T be the tetrahedron-group, let ρ_1, ρ_2 and ρ_3 be the three elements of T of order 2, and let ι be the neutral element of T. Further, let $H = \{\iota, \rho_1\}$ and $K = \{\iota, \rho_1, \rho_2, \rho_3\}$.

    (a) Show that $H \trianglelefteq K$.

    (b) Show that $K \trianglelefteq T$.

    Hint: Show that for each $\tau \in T$ and $\rho \in K$, $\tau\rho\tau^{-1}$ has order 2.

    (c) Show that H is not a normal subgroup of T.

33. Let G be a group.

    (a) Show that $Z(G) \leqslant G$.

    (b) Show that $Z(G) \trianglelefteq G$.

    (c) Show that if $H \leqslant Z(G)$, then $H \trianglelefteq G$.

34. Let T be the tetrahedron-group. What is the centre $Z(T)$ of T?

35. Let T be the tetrahedron-group, let ρ be an element of T of order 2 and let δ be an element of T of order 3. Further, let $H = \langle \rho \rangle$ and $K = \langle \delta \rangle$.

    (a) What is $H \cap K$?

    (b) Evaluate $|H|$, $|K|$ and $|HK|$.

    (c) Show that HK is not a subgroup of T.

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Office hours: Monday 3 pm–4 pm, Friday 1 pm–2 pm