Lorenz Halbeisen*

Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 8, 2002

You may download the ps-file from `http://www.math.berkeley.edu/~halbeis/4students/gt.html`

You don't have to hand in your solutions.

---

36. Let $S = \{p, q, r, s, t\}$ be a set and let "$\circ$" be the operation on S defined by the following multiplication table:

$\circ$	p	q	r	s	t
p	q	r	p	t	r
q	r	p	q	s	t
r	p	q	r	s	t
s	t	r	s	p	r
t	q	p	t	r	s

    (a) Does S has a neutral element?

    (b) Find the inverse of p, q, r, s and t, respectively.

    (c) Is $(S, \circ)$ a group? Why?

37. Let G be a group and let $H \leqslant G$.

    (a) How are the sets G/H and $H\backslash G$ defined?

    (b) What is the definition of a transversal for H in G?

    (c) What is the index of H in G?

38. Let G be a finite group and let $x \in G$.

    (a) What is $\langle x \rangle$?

    (b) What can you say about $|\langle x \rangle|$ and $|G|$?

    (c) What can you say about G, if $|\langle x \rangle| = |G|$?

    (d) Why is every group of order 43 abelian?

39. Let G be a group of order 150 and let $H \leqslant G$. What are the possible values for $|H|$?

40. (a) Give the definition of a normal subgroup N of a group G.

    (b) Give the definition of the centre $Z(G)$ of a group G.

    (c) Give the definition of a simple group.

    (d) Show that every group of order 53 is simple.

    (e) Give an example of a non-simple group of order 51.

---

*David Bates Building, Room 1014.

Office hours: Monday 3 pm–4 pm, Friday 1 pm–2 pm