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Group Theory (Module 210PMA208)

Department of Pure Mathematics

Week 9, 2002

You may download the ps-file from
http://www.math.berkeley.edu/~halbeis/4students/gt.html

Please hand in your solutions (stapled together with your full name on the first page) at the lecture on Monday, 22 April 2002.

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41. Let n be a positive integer. Show that

$$\begin{array}{rrcl} \varphi: & (\mathrm{GL}(n),\cdot\,) & \to & (\mathbb{R}^*,\cdot\,) \\ & A & \mapsto & \det(A) \end{array}$$

is a homomorphism, and decide for which n it is an isomorphism.

42. Show that

$$\begin{array}{rrcl} \varphi: & (\mathbb{C}^*,\cdot\,) & \to & \big(\{z \in \mathbb{C} : |z| = 1\},\cdot\,\big) \\ & z & \mapsto & \frac{z}{|z|} \end{array}$$

is a surjective homomorphism.

43. Let G be a group and let $x \in G$. Show that

$$\begin{array}{rrcl} \varphi_x: & G & \to & G \\ & a & \mapsto & xax^{-1} \end{array}$$

is an automorphism.

44. Let $m \in \mathbb{Z}$. Show that

$$\begin{array}{rrcl} \varphi: & (\mathbb{Z},+) & \to & (\mathbb{Z},+) \\ & x & \mapsto & mx \end{array}$$

is an endomorphism, and decide for which m it is an automorphism.

45. Let n be a positive integer. Define an injective homomorphism from C_n into D_n and one from C_n into D_{3n}.

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Office hours: Monday 3 pm–4 pm, Friday 1 pm–2 pm