... which implies that \(x_0\) is not an \(\varepsilon\)-minimal element...

... we have \(x_n \ni x_{n+1}\).

Now, let \(s_\alpha := f_0(M)\) and define \(F_{\alpha+1} := F_\alpha \cup \{\langle \alpha, s_\alpha \rangle\}\).

\[ ax_0^{k_0} \cdots x_i^{k_i} \text{ where...} \]

\[ x = \sum_{v \in B(x)} q_v^w \cdot v \]

\[ p_u \lor p_{-u} \text{ instead of } p_u \lor \neg p_{-u} \]

\[ \chi_A \cap \chi_B \supseteq \chi_{A \cup B} \]

Indeed, let \(g \in [z]^\omega\), and let \(\rho \neq \iota\) be such that \(\rho(y) = y\) and \(\rho\) induces a proper cycle in \([z]^\omega\) (i.e., the cycle starts and ends with \(y\) and the other points in the cycle are pairwise distinct).

... whenever \(\sigma\) has label \(\circlearrowright\), \(\varphi_n \sigma\) cannot get label \(\circlearrowright\).

... we have \(\pi a = \tau a\).

\[ f(s) := \{(m + l+1, s, 0), (m + l+1, s, 1)\} \]

... in \(\omega \setminus N_1\) instead of \(\omega \setminus (N_1 \cup N_2)\)

\[
\Psi_E : \{ S \subseteq A : \text{supp}(S) = E \} \rightarrow \mathcal{P}(\mathcal{P}(k))
\]

\[
S_0 \rightarrow \{ I \subseteq k : \exists a \in S_0 \left( \vartheta_E(a) = \{ \varphi_i(x) : i \in I \} \right) \}
\]

... \(\Psi_E\) maps \(S\) to \(\mathcal{P}(\mathcal{P}(k))\), and \(l < 2^\omega \) encodes the set \(\Psi_E(S)\).

\(g \in \omega^2\) (four times).

\[ x_0 := \bigcup \{ x \cap I_{2m} : m \in \omega \} \quad \text{and} \quad x_0 := \bigcup \{ x \cap I_{2m+1} : m \in \omega \}. \]

Now, since \(f(D') \subseteq D''\) and \(f(D'') \subseteq D' \cup (\omega \setminus D)\), this...

... but since \(f(I'_n) \subseteq I''_n \cup (\omega \setminus I_0)\) and \(f(I''_n) \subseteq I'_n \cup (\omega \setminus I_0)\), this is a contradiction to \(f(\mathcal{V}) = \mathcal{V}\). Thus, \(I_0 \notin \mathcal{V}\), which implies that \(I_\omega \in \mathcal{V}\). Now, for each \(n \in I_\omega\) there exists a least number \(m_n \in I_\omega\) such that there are \(k, k' \in \omega\) with \(f^k(m_n) = f^{k'}(n)\). Let

\[ I'_\omega := \left\{ n \in I_\omega : \exists k, k' \in \omega (f^k(m_n) = f^{k'}(n) \land k + k' \text{ is odd}) \right\} \]

and let \(I''_\omega := I_\omega \setminus I'_\omega\). Since the two sets \(I'_\omega\) and \(I''_\omega\) are disjoint and their union is \(I_\omega\), either \(I'_\omega\) or \(I''_\omega\) belongs to \(\mathcal{V}\), but not both. Furthermore, we get \(f(I'_n) \subseteq I''_n\) and \(f(I''_n) \subseteq I'_n\), which is again a contradiction to \(f(\mathcal{V}) = \mathcal{V}\).
\( V[G] = \{ \emptyset \} \)

\[ \forall (y_2, s_2) \in x_2 \forall q \in P \left( (q \geq s_2 \land q \Vdash \neg y_1 = y_2) \rightarrow q \perp r \right), \]

\( y[G] = \{ x[G] : \exists q \in G \left( (x, q) \in y \right) \} \)

and since \( p \in G \), for \( y = y[G] \) we get \( y \in V[G] \). Hence...

**Lemma 15.16.** If a forcing notion preserves cofinalities, then it preserves also cardinalities.

**Proof.** Since cofinalities are always cardinals, any forcing notion which preserves cardinalities must preserve cofinalities.

For the other direction,

Since \( p \in G \), for every \( \alpha \in \lambda \), \( G \cap D_\alpha \neq \emptyset \), and therefore, \( S[G](\alpha) \in Y_\alpha \).

If \( V \models ZFC \ldots \)

Let \( V \) be a model of \( ZFC \ldots \)

is equivalent to \( \psi \), free(\( \varphi_0 \)) \subseteq \ldots

\( h_{n,i}(\langle x_1, \ldots, x_i \rangle) := \mu \{ y \in V_{\alpha_{n+1}} : \forall x_{i+1} \in V_{\alpha_n} \exists y_{i+1} \cdots \forall x_k \in V_{\alpha_n} \exists y_k \ldots \}

for each \( a \in A \), \( \{ \alpha \in G : \alpha a = a \} \in \mathcal{F} \)

Let \( G \) be the group generated by automorphisms of \( C_\omega \) of the form \( \alpha_{n_1}, n_0 \), i.e.,

\[ \mathcal{G} = \langle \alpha_{n_1}, n_0 : F \in \text{fin}(\omega) \land n_0 \in \omega \rangle. \]

\( H_\omega \)

\( \delta_{\omega_1} := \bigcup_{i \in \omega_1} \delta_i \)

---

**Minor Corrections and Improvements**

Let \( \varphi, \varphi_1, \varphi_2, \varphi_3 \), and \( \psi \ldots \)

.. is equal to the formula \( \forall \nu \varphi_j \), where \( \nu \) is a variable which does not occur free in any non-logical axiom of \( T \).

subset instead of subsets

\( \ldots z^m \leq \text{seq}(m) \ldots \)
... \forall \alpha \in \omega, \text{seq}(\alpha) \leq \text{seq}(\beta) \ldots 

which shows that \( V_{\text{eq}} \) can be well-ordered in the case when \( \alpha_0 \) is a successor ordinal.

\( m \cdot 2^{\aleph_0} \leq \text{seq}(m + \aleph_0) \ldots \)

\( \text{seq}(m + \aleph_0) \leq \text{seq}(m) \ldots \)

\( \text{seq}(m + \aleph_0) = \omega \ldots \)

\( \text{seq}(m + \aleph_0) = \omega \ldots \)

\( \text{seq}(m + \aleph_0) = \omega \ldots \)

\( \sigma_0(x_0) = \sigma(x_0) \) and therefore \( \sigma_0^{-1} \sigma_0(x_0) = x_0 \). Consequently we have \( \sigma_0^{-1} \sigma_0 = \vartheta_0 \), and therefore \( \rho = \sigma_0 \vartheta_0 \sigma_0^{-1} \). Thus, since \( \rho \) induces a proper cycle, this implies \( y \in \{x_0, \ldots, x_k\} \).

The Ordered Mostowski Model instead of “Ordered Mostowski Models”.

Fraïssé-limit

Fraïssé-limit

\( \Psi : \mathcal{P}(A) \ldots \)

\( J \) are arbitrary finite, disjoint subfamilies.

In other words, \( \text{MA}(\kappa) \) holds for each cardinal \( \kappa < c \)

\[ \text{up}(x, y) = \{ \langle x, 0 \rangle, \langle y, 0 \rangle \} \]

and

\[ \text{op}(x, y) = \{ \langle \{ \langle x, 0 \rangle \}, 0 \rangle, \langle \{ \langle x, 0 \rangle, \langle y, 0 \rangle \}, 0 \rangle \} \].

Replace everywhere \( G \) with \( G \hat{} \), and cancel in the index the definition of \( G \).

In order to show the second part of this proof \( (G \) is \( \mathbb{P} \)-generic) one needs FACT 15.7.

\( \text{collapses} \; \kappa \) [bold] and \( \text{preserves} \; \kappa \) [bold]

This is because whenever \( q_1 \vdash \mathcal{S} \models \gamma_1 \) and \( q_2 \vdash \mathcal{S} \models \gamma_2 \), where \( \gamma_1 \neq \gamma_2 \), then \( q_1 \perp q_2 \).

Replace \( p \) with \( p_0 \) on line 6, 7, 9, 10, 15.

\( \text{countable union of at most countable sets of ordered pairs} \ldots \)

\( \text{countable union of at most countable sets of ordered pairs} \ldots \)

\( \text{refine} \) the construction in the proof of (a). By \( \ldots \)

\( \text{is a countable transitive model in} \; V, \; N[G] \models \Phi_0 \), and if \( p_0 \vdash \varphi \), then \( N[G] \models \varphi \).

\( \text{then} \; N[G] \models \varphi_0 + \varphi \).

Since \( \mathcal{U} \) is generated by \( \mathcal{U} \), for each \( n \in \omega \) there is an \( x_n' \in \mathcal{U} \) such that \( x_n' \subseteq x_n \). Then define \( A := \{ f(x_n') : n \in \omega \} \) and notice that \( y \subseteq x_n' \subseteq x_n \).
A general form of the $\Delta$-System-Lemma (see Kunen, Thm. 1.6, p. 49) is needed here.

$\ldots \leq \omega_2 \cdot \omega_2 \ldots$

$\ldots$ $P$-point—and in particular every Ramsey ultrafilter—in $\ldots$

$\ldots$ $P$-point in $V[G_\delta]$, for some $\delta \in \omega_2$. 