CORRECTIONS AND IMPROVEMENTS
19 February 2020

page 35, line 10 . . . which implies that \( x_0 \) is not an \( \varepsilon \)-minimal element . . .

page 48, line 13 . . . we have \( x_n \not\in x_{n+1} \).

page 118, line 3 Now, let \( s_\alpha := f_0(M) \) and define \( F_{\alpha+1} := F_\alpha \cup \{ (\alpha, s_\alpha) \} \).

page 141, line 21 \( ax_0^k \cdots x_i^k \) where . . .

page 141, line -4 \( x = \sum_{v \in B(x)} q_i^v \cdot v \)

page 152, line -16 \( p_u \lor p_{-u} \) instead of \( p_u \lor \neg p_{-u} \)

page 154, line 11 \( \chi_A \cap \chi_B \supseteq \chi_{A \cup B} \)

page 185, line 2 ff. Indeed, let \( y \in [z]^\ast \), and let \( \rho \neq \iota \) be such that \( \rho(y) = y \) and \( \rho \) induces a proper cycle in \([z]^\ast\) (i.e., the cycle starts and ends with \( y \) and the other points in the cycle are pairwise distinct).

page 186, line 2 . . . whenever \( \sigma \) has label \( 1 \), \( \varphi_n \sigma \) cannot get label \( \circ \).

page 208, line 4 . . . we have \( \pi a = \tau a \).

page 210, line 7 \( f(s) := \{(m + l + 1, s, 0), (m + l + 1, s, 1)\} \)

page 213, line 9 . . . in \( \omega \setminus N_1 \) instead of \( \omega \setminus (N_1 \cup N_2) \)

page 215, line 20f. \[ \Psi_E : \{ S \subseteq A : \text{supp}(S) = E \} \longrightarrow \mathcal{P}(\mathcal{P}(k)) \]

\[ S_0 \longrightarrow \{ I \subseteq k : \exists a \in S_0 \left( \varnothing_E(a) = \{ \varnothing_i(x) : i \in I \} \right) \} \]

page 215, line -7 . . . \( \Psi_E \) maps \( S \) to \( \mathcal{P}(\mathcal{P}(k)) \), and \( l < 2^x \) encodes the set \( \Psi_E(S) \) . . .

page 231, line 10 ff. \( g \in \omega^2 \) (four times).

page 267, line 18 \( x_0 := \bigcup \{ x \cap I_{2m} : m \in \omega \} \) and \( x_0 := \bigcup \{ x \cap I_{2m+1} : m \in \omega \} \).

page 279, line 4 Now, since \( f(D') \subseteq D'' \) and \( f(D'') \subseteq D' \cup (\omega \setminus D) \), this . . .

page 279, line 12 ff. . . . but since \( f(I_0') \subseteq I_0' \cup (\omega \setminus I_0) \) and \( f(I_0'') \subseteq I_0' \cup (\omega \setminus I_0) \), this is a contradiction to \( f(\mathcal{U}) = \mathcal{V} \). So, \( I_0 \not\in \mathcal{U} \), which implies that \( I_\omega \in \mathcal{U} \). Now, for each \( n \in I_\omega \) there exists a least number \( m_n \in I_\omega \) such that there are \( k, k' \in \omega \) with \( f^k(m_n) = f^{k'}(n) \). Let \( I_\omega' := \{ n \in I_\omega : \exists k, k' \in \omega (f^k(m_n) = f^{k'}(n) \land k + k' \text{ is odd}) \} \)

and let \( I_\omega'' := I_\omega \setminus I_\omega' \). Since the two sets \( I_\omega' \) and \( I_\omega'' \) are disjoint and their union is \( I_\omega \), either \( I_\omega' \) or \( I_\omega'' \) belongs to \( \mathcal{U} \), but not both. Furthermore, we get \( f(I_0') \subseteq I_0'' \) and \( f(I_0'') \subseteq I_0'' \), which is again a contradiction to \( f(\mathcal{U}) = \mathcal{V} \).
...for any uncountable set...

Now we show that $|\mathcal{D}| < \mathfrak{c}$ cannot...

$V[G] = \{\emptyset\}$

$\forall(y_2, s_2) \in x_2 \forall q \in P \left( (q \geq s_2 \land q \not\rightarrow_p y_1 = y_2) \rightarrow q \perp r \right)$,

...and since $y[G] = \{x[G] : \exists q \in G (x, q) \in y\}$

...and since $p \in G$, for $y = y[G]$ we get $y \in V[G]$. Hence...

Four times $\bigcup G$ instead of just $G$.

LEMMA 15.16. If a forcing notion preserves cofinalities, then it preserves also cardinalities.

Proof. Since cofinalities are always cardinals, any forcing notion which preserves cardinals must preserve cofinalities.

For the other direction,

Since $p \in G$, for every $\alpha \in \lambda$, $G \cap D_\alpha \neq \emptyset$, and therefore, $S[G](\alpha) \in Y_\alpha$.

If $V \not\models \text{ZFC}$...

...Let $V$ be a model of ZFC...

...is equivalent to $\psi$, free($\varphi_0$) $\subseteq$ ...

...reflects $\psi$.

$h_{n,i}(x_1, \ldots, x_i) := \mu\{y \in V_{\alpha_{n+1}} : \forall x_{i+1} \in V_{\alpha_n} \exists y_{i+1} \cdots \forall x_k \in V_{\alpha_n} \exists y_k \ldots$ for each $a \in A$, $\{\alpha \in G : \alpha a = a\} \in F$

Let $G$ be the group generated by automorphisms of $C_\omega$ of the form $\alpha_{\pi, n_0}$, i.e.,

$G = \langle \alpha_{\pi, n_0} : F \in \text{fin}(\omega) \land n_0 \in \omega \rangle$.

...corresponds an automorphism $\alpha_\pi$ of $\mathbb{P}$ by stipulating

$\alpha_\pi p(\pi\langle\bar{a}, \xi, \eta\rangle) := p(\bar{a}, \xi, \eta)$,

$\{H : H \in \mathcal{F}_0\} \cup \{\text{fix}_{\tilde{g}}(E) : E \in \text{fin}(A \times \kappa)\}$.

...i.e., for every $\sigma \in \text{sym}_{\mathcal{G}_0}(a)$, $\tilde{\sigma} \subseteq \text{sym}_{\tilde{g}}(\tilde{a})$

$H_{\omega_1}$
MINOR CORRECTIONS AND IMPROVEMENTS

page 14, line 16 Let $\varphi, \varphi_1, \varphi_2, \varphi_3,$ and $\psi \ldots$

page 16, line -6 . . . is equal to the formula $\forall \nu \varphi_j$, where $\nu$ is a variable which does not occur free in any non-logical axiom of $T$.

page 41, line 9 subset instead of subsets

page 120, line 1 . . . $2^m \leq \text{seq}(m) \ldots$

page 120, line 14 . . . $2^m \cdot 2^{\aleph_0} \leq \text{seq}(m + \aleph_0) \ldots$

page 143, line -2 f. which shows that $V_{\omega_1}$ can be well-ordered in the case when $\alpha_0$ is a successor ordinal.

page 154, line -16 add a space: } Notice

page 185, line 3 ff. . . is as above. So, $\rho \sigma_y(x_0) = \sigma_y(x_0)$ and therefore $\sigma_y^{-1} \rho \sigma_y(x_0) = x_0$. Consequently we have $\sigma_y^{-1} \rho \sigma_y = \sigma^n$, and therefore $\rho = \sigma_y \sigma^n \sigma_y^{-1}$. Thus, since $\rho$ induces a proper cycle, this implies $y \in \{x_0, \ldots, x_k\}$.

page 198, line -9 The Ordered Mostowski Model instead of “Ordered Mostowski Models”.

page 211, line -3 Fraïssé-limit

page 213, line 18 Fraïssé-limit

page 215, line -11 $\Psi : \mathcal{P}(A) \ldots$

page 231, line -9 . . . $J$ are arbitrary finite, disjoint subfamilies . .

page 323, line 2, . . . $(P, \leq)$

page 324, line -7 $\mathcal{D} \subseteq \mathcal{P}(P)$

page 325, line 17 In other words, $\text{MA}(\kappa)$ holds for each cardinal $\kappa \leq \kappa_c$

page 343, line -6 ff.

$$\text{up}(x, y) = \{\langle x, 0 \rangle, \langle y, 0 \rangle\}$$

and

$$\text{op}(x, y) = \{\langle \{\langle x, 0 \rangle\}, 0 \rangle, \langle \{\langle x, 0 \rangle, \langle y, 0 \rangle\}, 0 \rangle\}.$$
page 361, line 1  **collapses** $\kappa$ [bold] and **preserves** $\kappa$ [bold]

page 362, line 13  This is because whenever $q_1 \Vdash P S(\alpha) = \gamma_1$ and $q_2 \Vdash P S(\alpha) = \gamma_2$, where $\gamma_1 \neq \gamma_2$, then $q_1 \perp q_2$.

page 362, line 6 ff.  Replace $p$ with $p_0$ on line 6, 7, 9, 10, 15.

page 364, line 17 f.  . . . countable union of at most countable sets of ordered pairs . . .

page 364, line 17 f.  . . . countable union of at most countable sets of ordered pairs . . .

page 372, line -5  . . . (b), we refine the construction in the proof of (a). By . . .

page 378, line 5  . . . is a countable transitive model in $V$, $N[G] \models \Phi_0$, and if $p_0 \Vdash \varphi$, then $N[G] \models \varphi$.

page 378, line 20  . . . then $N[G] \models \Phi_0 + \varphi$.

page 465, line 2 ff.  Since $\hat{\mathcal{U}}$ is generated by $\mathcal{U}$, for each $n \in \omega$ there is an $x'_n \in \mathcal{U}$ such that $x'_n \subseteq x_n$. Then define $A := \{ f(x'_n) : n \in \omega \}$ and notice that $y \subseteq^* x'_n \subseteq x_n$.

page 492, line -11  A general form of the $\Delta$-System-Lemma (see Kunen, Thm. 1.6, p. 49) is needed here.

page 493, line 3  . . . $\leq \omega_2 \cdot \omega_2$ . . .

page 493, line -8  . . . $P$-point—and in particular every Ramsey ultrafilter—in . . .

page 551, line -6  . . . $P$-point in $V[G|\delta]$, for some $\delta \in \omega_2$. 