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Bio-inspired sensing and imaging

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- Mimic electrolocation by weakly electrical fish and echolocation by bats.
- Enhance the resolution, the robustness, and the specificity of tissue property imaging modalities.



Long-nosed elephant fish

Long-eared bat

- Biological vision: two types of retina-brain pathways in the visual system.
  - Transient magno-cellular pathway and the sustained parvo-cellular pathway.
  - Magno-system: sensitive to changes and movements; detect dangers that arise in the peripheral vision.



**Biological vision** 

- Key concepts:
  - Resolution: smallest distance between two point reflectors that can be resolved; limited by half the operating wavelength.
  - Robustness: stability of the image formation with respect to model uncertainty and medium and electronic noises.
  - Specificity: physical nature (for tumors: benign or malignant).





### Tissue property imaging

- Tissue property imaging: electromagnetic and elastic waves play a key role in visualizing contrast information on the electrical, optical, mechanical properties of tissues.
- Tissue contrasts:
  - Highly sensitive to physiological and pathological tissue status.
  - Depend on the cell organization and composition.
  - Overall parameters, averaged in space over many cells.
- Recognize the microscopic cell organization and composition from measurements at the macroscopic level.



### Tissue property imaging

- Diagnosis and staging of cancer disease.
- Help surgeons to make sure they removed everything unwanted around the margin of the cancer tumor.
- Perform biopsy in the operating room.





# Tissue property imaging

- Electrical tissue properties:
  - Electrical conductivity: tissue's ability to transport charges;
  - Electrical permittivity (dielectric constant): tissue's ability to trap or to rotate molecular dipoles; determined by the polarization under an external electric field;
  - Frequency-dependent or dispersive; anisotropic;
  - Capacitive effect generated by the cell membrane structure;
  - Macroscopic parameters; represent the electrical properties of the tissue averaged in space over many cells.

# Bio-inspired dictionary matching based approach

- Electrolocation for weakly electric fish<sup>1</sup>:
  - Electric organ: generates a stable, high-frequency, weak electric field.
  - Electroreceptors: measure the transdermal potential modulations caused by a nearby target.
  - Nervous system: locates the target, perceives its shape, determines its physical nature.



<sup>1</sup>with T. Boulier, J. Garnier, and H. Wang, Proc. Natl. Acad. Sci., 2014. ≥ → ૧. Bio-inspired sensing and imaging Habib Ammari

# Shape perception

Mechanism for mimicking shape perception:

- Form an image from the perturbations of the field due to targets.
- Identify and classify the target, knowing by advance that it belongs to a learned dictionary of shapes.
  - Extract the features from the data.
  - Construct invariants with respect to rigid transformations and scaling.
  - Compare the invariants with precomputed ones for the dictionary.
- Biological targets: frequency dependent electrical properties (capacitive effect generated by the cell membrane structure).
- $\Rightarrow$  Spectroscopic measurements of the target's polarization tensor.

• Wave-type electric signal:  $f(x, t) = f(x) \sum_{n} a_n e^{in\omega_0 t}$ ;  $\omega_0$ : fundamental frequency.



• Skin: very thin ( $\delta \sim 100 \mu$ m) and highly resistive ( $\sigma_s/\sigma_0 \sim 10^{-2}$ );  $\sigma_b/\sigma_0 \sim 10^2$  (highly conductive).



- Target  $D = z + \delta'B$ ; z: location;  $\delta'$ : characteristic size of the target;  $k(\omega) = (\sigma(\omega) + i\omega\varepsilon(\omega))/\sigma_0$ ; k,  $\sigma$ , and  $\varepsilon$ : the admittivity, the conductivity, and the permittivity of the target;  $\omega_n = n\omega_0$ : the probing frequency.
- $u_n$ : the electric potential field generated by the fish:

$$\begin{cases} \Delta u_n = f, & x \in \Omega, \\ \nabla \cdot (1 + (k(\omega_n) - 1)\chi(D))\nabla u_n = 0, & x \in \mathbb{R}^2 \setminus \overline{\Omega}, \\ \frac{\partial u_n}{\partial \nu} \bigg|_{-} = 0, \quad [u_n] = \frac{\xi}{2} \frac{\partial u_n}{\partial \nu} \bigg|_{+} & x \in \partial\Omega, \\ |u_n(x)| = O(|x|^{-1}), \quad |x| \to \infty. \end{cases}$$

- $\xi := \delta \sigma_0 / \sigma_s$  effective thickness.
- $\lambda(\omega_n) = (k(\omega_n) + 1)/(2(k(\omega_n) 1)).$

- Dipole approximation:  $u_n(x) U(x) \simeq \mathbf{p} \cdot \nabla G(x-z)$ .
  - G: Green's function associated to Robin boundary conditions.
  - Dipole moment  $\mathbf{p} = \underbrace{\mathcal{M}(\lambda(\omega_n), D)}_{\mathcal{V}} \nabla U(z).$

#### Polarization tensor

• Neumann-Poincaré operator:

$$\mathcal{K}_{D}^{*}[\varphi](x) = \frac{1}{2\pi} \int_{\partial D} \frac{\langle x - y, \nu_{x} \rangle}{|x - y|^{2}} \varphi(y) \, ds(y) \,, \quad x \in \partial D.$$

• 
$$M(\lambda(\omega_n), D) = \int_{\partial D} x(\lambda(\omega_n)I - \mathcal{K}_D^*)^{-1}[\nu](x) ds(x).$$



- *K*<sup>\*</sup><sub>D</sub>: compact operator on *L*<sup>2</sup>(∂*D*); Spectrum of *K*<sup>\*</sup><sub>D</sub> lies in (-<sup>1</sup>/<sub>2</sub>, <sup>1</sup>/<sub>2</sub>] (Kellog).
- Spectral decomposition formula in  $H_0^{-1/2}(\partial D)$ ,

$$\mathcal{K}_D^*[\psi] = \sum_{j=0}^{\infty} \lambda_j(\psi, \varphi_j)_{\mathcal{H}^*} \varphi_j.$$

- $(\lambda_j, \varphi_j), j = 0, 1, 2, \ldots$ : eigenvalue and normalized eigenfunction pair of  $\mathcal{K}_D^*$  in  $\mathcal{H}^*(\partial D); \lambda_j \in (-\frac{1}{2}, \frac{1}{2}]$  and  $\lambda_j \to 0$  as  $j \to \infty$ ;
- $\mathcal{H}^*(\partial D) = H_0^{-\frac{1}{2}}(\partial D)$  equipped with

 $(u, v)_{\mathcal{H}^*} = -(u, \mathcal{S}_D[v])_{-\frac{1}{2}, \frac{1}{2}}; \quad \mathcal{S}_D : \text{single layer potential.}$ 

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Space-frequency response matrix: (V<sup>n</sup><sub>sr</sub>)<sub>rn</sub>

$$V_{sr}^{n} = \left( \left. \frac{\partial u_{n}}{\partial \nu}(x_{r}) \right|_{+} - \left. \frac{\partial U}{\partial \nu}(x_{r}) \right|_{+} \right),$$

 $x_s$ : position of the electric organ;  $(x_r)$ : receptors on the skin of the fish.

- Space-frequency location search algorithm.
- Movement: Fish takes measurement at different positions around the target ⇒ can use only one frequency.



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• Dipole approximation:

$$V_{sr}^n \simeq -\nabla U(z) \cdot \overbrace{\mathcal{M}(\lambda(\omega_n), D)}^{\propto l} \cdot \left( \nabla \frac{\partial \mathcal{G}}{\partial \nu_x}(x_r - z) \right);$$

•  $z^{s}$  in the search domain; Vector field  $g(z^{s})$  given by

$$\left(\nabla U(z^{S}) \cdot \nabla \left(\frac{\partial G}{\partial \nu_{x}}\right)(x_{1}-z^{S}),\ldots,\nabla U(z^{S}) \cdot \nabla \left(\frac{\partial G}{\partial \nu_{x}}\right)(x_{L}-z^{S})\right)^{T};$$

• Subspace imaging functional:

$$\mathcal{I}(z^{S}):=\frac{1}{|(I-P)g(z^{S})|};$$

*P*: orthogonal projection onto the first singular vector of  $(V_{sr}^n)_{rn}$ ;

•  $\mathcal{I}(z^{S})$ : large peak at  $z^{S} = z$ .



•  $\sigma, \varepsilon$ : determined by minimizing a quadratic misfit functional.

Dictionary matching based approach



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- Multi-frequency approach:  $\omega \mapsto M(\lambda(\omega), D)$ .
  - Invariance with respect to translation, rotation, and scaling.
  - τ<sub>j</sub>(ω): eigenvalues of ℑm M(λ(ω), D); ω<sub>∞</sub>: highest probing frequency. Plot

$$\omega \mapsto \frac{\tau_j(\omega)}{\tau_j(\omega_\infty)},$$

for j = 1, ..., d.

# Dictionary matching based approach



Probability of detection in terms of the noise level. Stability of classification based on differences between ratios of eigenvalues of  $\Im m M(\lambda(\omega), D)$ .

Nonbiological targets (frequency-independent electrical parameters):



F. Boyer

• Use multipolar approximation:

$$u_n(x) - U(x) \simeq \sum_{\alpha,\beta} \partial^{\alpha} G(x-z) M_{\alpha\beta}(\lambda, D) \partial^{\beta} U(z).$$

•  $M_{\alpha\beta}(\lambda, D)$ : high-order polarization tensors.

$$M_{\alpha\beta}(\lambda,D) := \int_{\partial D} x^{\beta} (\lambda I - \mathcal{K}_{D}^{*})^{-1} [\partial x^{\alpha} / \partial \nu](x) \, ds(x)$$

Properties of high-order polarization tensors:

- Recover high-frequency information on the shape;
- Separate topology;
- Determine uniquely the shape and the material parameter.



- Positivity and symmetry properties on harmonic coefficients; optimal bounds.
- Harmonic coefficients:

$$(x_1+ix_2)^m = \sum_{|\alpha|=m} a^m_{\alpha} x^{\alpha} + i \sum_{|\beta|=m} b^m_{\beta} x^{\beta}.$$

- Translation, rotation, and scaling formulas.
- Construct shape descriptors invariant with respect to translation, rotation, and scaling.



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- Reconstruction of high-order polarization tensors from the data by a least squares method.
- Instability:

 $M_{lphaeta}(k,D)=O(|D|^{|lpha|+|eta|+d-2}), |\partial^{lpha}G(x-z)|=O(|x|^{-|lpha|})(|x|
ightarrow+\infty).$ 

- Resolving power= number of high-order polarization tensors reconstructed from the data: depends on the signal-to-noise ratio (SNR) in the data.
- SNR =  $\epsilon^2$ /standard deviation of the measurement noise (Gaussian).
- Formula for the resolving power *m* as function of the SNR:

$$(m\epsilon^{1-m})^2 = \text{SNR}.$$



Stability of classification based on Shape Descriptors.

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# Spectroscopic electrical tissue property imaging

- Differentiate between normal, pre-cancerous and cancerous tissues from electrical measurements at tissue level.
- Frequency dependence of the (anisotropic) homogenized admittivity:  $\omega \mapsto K^*(\omega).$
- Relaxation times:
  - 1/ arg max<sub>ω</sub> eigenvalues of ℑm K<sup>\*</sup>(ω);
  - Classification: invariance properties;
  - Measure of anisotropy: ratios of the eigenvalues of  $\Im m K^*(\omega)$ .



#### Spectroscopic electrical tissue property imaging

The effective admittivity of a periodic dilute suspension<sup>2</sup>:

$$K^* = k_0 \left( I + fM \left( I - \frac{f}{2}M \right)^{-1} \right) + o(f^2).$$

- *M*: membrane polarization tensor

$$M = -\left(\xi \int_{\partial \widetilde{D}} \nu_j \left(I + \xi L_{\widetilde{D}}\right)^{-1} [\nu_i](y) ds(y)\right)_{i,j=1,2}$$

• 
$$L_{\widetilde{D}}[\varphi](x) = \frac{1}{2\pi} \text{p.v.} \int_{\partial \widetilde{D}} \frac{\partial^2 \ln |x - y|}{\partial \nu(x) \partial \nu(y)} \varphi(y) ds(y), \quad x \in \partial \widetilde{D}.$$

 $^2$  with J. Garnier, L. Giovangigli, W. Jing, and J.K. Seo, J. Math. Pures Appl., 2016.

## Spectroscopic electrical tissue property imaging

- Properties of the membrane polarization tensor:
  - *M*: symmetric; invariant by translation;
  - $M(sC,\xi) = s^2 M(C,\frac{\xi}{s})$  for any scaling parameter s > 0.
  - $M(\mathcal{RC},\xi) = \mathcal{R}M(\mathcal{C},\xi)\mathcal{R}^t$  for any rotation  $\mathcal{R}$ .
  - Sm M is positive and its eigenvalues, λ<sub>1</sub> ≥ λ<sub>2</sub>, have one maximum with respect to ω.
- Relaxation times for the arbitrary-shaped cells:

 $rac{1}{ au_i} := rg\max_{\omega} \lambda_i(\omega).$ 

- $(\tau_i)_{i=1,2}$ : invariant by translation, rotation and scaling.
- Concentric circular-shaped cells: Maxwell-Wagner-Fricke formula (λ<sub>1</sub> = λ<sub>2</sub>).
- Nondilute regime: Assume f known ⇒ Classification based on relaxation times.

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#### Bats

- Dictionary matching based approach for target classification in echolocation<sup>3</sup>.
- $u^i$ : incident wave;  $\kappa$ : bulk modulus;  $\rho$ : density. Helmholtz equation:

$$\begin{cases} \nabla \cdot \left( \chi(\mathbb{R}^2 \setminus \bar{D}) + \frac{1}{\rho} \chi(D) \right) \nabla u + \omega^2 \left( \chi(\mathbb{R}^2 \setminus \bar{D}) + \frac{1}{\kappa} \chi(D) \right) u = 0, \\ u^s := u - u^i \text{ satisfies the outgoing radiation condition.} \end{cases}$$



<sup>3</sup>with P. Tran and H. Wang, SIAM J. Imag. Sci., 2014.

• Scattering coefficients<sup>4</sup>:

$$W_{mn}(D,\kappa,\rho,\omega) = \int_{\partial D} \psi_m(y) J_n(\omega|y|) e^{-in\theta_y} ds(y).$$

$$\underbrace{J_m(\omega|x|)e^{im\theta_x}}_{J_m(\omega|x|)} + \mathcal{S}^{\omega}_D[\psi_m] \quad x \in \mathbb{R}^d \setminus \overline{D};$$

cylindrical wave

•  $J_m$ : Bessel function.

<sup>4</sup>with M. Lim, H. Kang, H. Lee, Comm. Math. Phys., 2014. ( ) + ( )

### Bats

#### Properties of the scattering coefficients:

• *W<sub>mn</sub>* decays rapidly:

$$|W_{mn}| \lesssim \frac{C^{|m|+|n|}}{|m|^{|m|}|n|^{|n|}}, \ m,n \in \mathbb{Z}.$$

• For any 
$$z\in \mathbb{R}^2, heta\in [0,2\pi), s>0$$
,

• Translation:

$$W_{mn}(D^{\mathbf{z}}) = \sum_{m',n'\in\mathbb{Z}} J_{n'}(\omega|\mathbf{z}|) J_{m'}(\omega|\mathbf{z}|) e^{i(m'-n')\theta_{\mathbf{z}}} W_{m-m',n-n'}(D);$$

• Rotation:

$$W_{mn}(D^{\theta}) = e^{i(m-n)\theta} W_{mn}(D);$$

• Scaling:

$$W_{mn}(D^{s},\omega)=W_{mn}(D,s\omega).$$

• Scattering amplitude:

$$u^{s}(x) = -ie^{-\frac{\pi i}{4}} \frac{e^{i\omega|x|}}{\sqrt{8\pi\omega|x|}} A_{\infty}[D,\kappa,\rho,\omega](\theta,\theta') + o(|x|^{-\frac{1}{2}}),$$

 $|x| \to \infty; \; u^i :$  plane wave;  $\theta, \; \theta' :$  incident and scattered directions.

• Graf's formula:

$$A_{\infty}[D,\kappa,\rho,\omega](\theta,\theta') = \sum_{n,m\in\mathbb{Z}} (-i)^n i^m e^{in\theta'} W_{nm}(D,\kappa,\rho,\omega) e^{-im\theta}.$$

#### Feature extraction:

- *V*: measurements; *W*: features; L: linear operator.
- Extract W by solving a least-squares method

$$\mathbf{W} = \underset{\mathbf{W}}{\operatorname{arg\,min}} \|\mathbf{L}(\mathbf{W}) - \mathbf{V}\|.$$

- L: ill-conditioned.
- Formula for the resolving power as function of the SNR: Maximum resolving order K satisfies

$$\mathcal{K}^{\mathcal{K}+1/2} = \mathcal{C}(\omega)$$
SNR.

#### Bats



Shape descriptor matching in a multi-frequency dictionary.

# Acoustic cloaking

- Make a target invisible when probed by acoustic waves.
- Cloaking: scattering coefficient cancellation<sup>5</sup>:
  - Small layered object with vanishing first-order scattering coefficients.
  - Transformation optics:

$$(F_{\eta})_*[\phi](y) = \frac{DF_{\eta}(x)\phi(x)DF_{\eta}(x)^t}{\det(DF_{\eta}(x))}, \quad x = F_{\eta}^{-1}(y).$$

 Change of variables F<sub>η</sub> sends the annulus [η, 2η] onto a fixed annulus.

<sup>5</sup>with M. Lim, H. Kang, H. Lee, Comm. Math. Phys., 2014.  $\leftarrow = \rightarrow$ 

### Acoustic cloaking

• Scattering cross-section:

$$Q^s[D,\kappa,
ho,\omega]( heta'):=\int_0^{2\pi} \left| A_\infty[D,\kappa,
ho,\omega]( heta, heta') 
ight|^2 d heta.$$

• Scattering coefficients vanishing structures of order *N*:

$$Q^{s}\Big[D,(F_{\eta})_{*}(\rho\circ\Psi_{\frac{1}{\eta}}),(F_{\eta})_{*}(\kappa\circ\Psi_{\frac{1}{\eta}}),\omega\Big](\theta')=o(\eta^{4N}).$$

 $\eta$ : size of the small object; *N*: number of layers;  $\Psi_{1/\eta}(x) = (1/\eta)x$ .

- Anisotropic density and bulk modulus distributions.
- Invisibility at  $\omega \Rightarrow$  invisibility at all frequencies  $\leq \omega$ .

# Acoustic cloaking

• Cloaking: scattering coefficient cancellation



Cancellation of the scattered field and the scattering cross-section: 4 orders of magnitude (with wavelength of order 1,  $\eta = 10^{-1}$ , and N = 1).

- Ultrasound-modulated optical tomography
- Cross-correlation techniques.
- Hybrid imaging modality: one single imaging system based on the combined use of different imaging modalities.



• Acoustically modulated optical tomography<sup>6</sup>:



• Record the variations of the light intensity on the boundary due to the propagation of the acoustic pulses.

<sup>6</sup>with E. Bossy, J. Garnier, L. Nguyen, L. Seppecher; Proc. AMS, 2014. Bio-inspired sensing and imaging Habib Ammari

 g: the light illumination; a: optical absorption coefficient; *I*: extrapolation length. Fluence Φ (in the unperturbed domain):

$$\begin{cases} -\Delta \Phi + a\Phi = 0 \text{ in } \Omega, \\ I \frac{\partial \Phi}{\partial \nu} + \Phi = g \text{ on } \partial \Omega \end{cases}$$

- Acoustic pulse propagation:  $a \rightarrow a_u(x) = a(x + u(x))$ .
- Fluence  $\Phi_u$  (in the displaced domain):

$$\begin{cases} -\Delta \Phi_u + a_u \Phi_u = 0 \text{ in } \Omega, \\ I \frac{\partial \Phi_u}{\partial \nu} + \Phi_u = g \text{ on } \partial \Omega. \end{cases}$$

- *u*: thin spherical shell growing at a constant speed; *y*: source point; *r*: radius.
- Cross-correlation formula:

$$M(y,r) := \int_{\partial\Omega} \left( \frac{\partial \Phi}{\partial \nu} \Phi_u - \frac{\partial \Phi_u}{\partial \nu} \Phi \right) = \int_{\Omega} (a_u - a) \Phi \Phi_u \approx \underbrace{\int_{\Omega} u \cdot \nabla a |\Phi|^2}_{Taylor + Born}.$$

- Helmholtz decomposition:  $\Phi^2 \nabla a = \nabla \psi + \nabla \times A$ .
- Spherical Radon transform:  $\nabla \psi = -\frac{1}{c} \nabla \mathcal{R}^{-1} \left[ \int_0^r \frac{M(y,\rho)}{\rho^{d-2}} d\rho \right].$
- System of nonlinearly coupled elliptic equations:  $\nabla \cdot \Phi^2 \nabla a = \Delta \psi$  and  $-\Delta \Phi + a \Phi = 0$ .
- Fixed point and Optimal control algorithms.
- Convergence result for the fixed point scheme provided that  $\|\Delta\psi\|_{L^{\infty}(\Omega)}$ : small.
- Convergence result for the optimal control algorithm assuming a good initial guess.

- Reconstruction for a realistic absorption map: proof of convergence for highly discontinuous absorption maps (bounded variation)<sup>7</sup>.
- Minimal regularity assumption on the absorption coefficient:  $a \in SBV^{\infty}$ .





<sup>7</sup>with L. Nguyen and L. Seppecher, J. Funct. Anal., 2014,  $\triangleright$ 

- Gold nano-particles: accumulate selectively in tumor cells; bio-compatible; reduced toxicity.
- Detection: localized enhancement in radiation dose (strong scattering).
- Ablation: localized damage (strong absorption).
- Functionalization: targeted drugs.



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- D: nanoparticle; ε<sub>c</sub>(ω): complex permittivity of D; ε<sub>m</sub> > 0: permittivity of the background medium;
- Permittivity contrast:  $\lambda(\omega) = (\varepsilon_c(\omega) + \varepsilon_m)/(2(\varepsilon_c(\omega) \varepsilon_m)).$
- $G_{k_m}$ : outgoing fundamental solution to  $\Delta + k_m^2$ ;  $k_m := \omega/\sqrt{\varepsilon_m}$ .
- Quasi-static far-field approximation<sup>8</sup>:  $|D| \rightarrow 0$ ,

$$u^s = -M(\lambda(\omega), D) 
abla_z G_{k_m}(x-z) \cdot 
abla u^i(z) + O(rac{|D|^{3/2}}{\operatorname{dist}(\lambda(\omega), \sigma(\mathcal{K}_D^*))}).$$

• Spectral decomposition: (1, m)-entry

$$M_{l,m}(\lambda(\omega),D) = \sum_{j=1}^{\infty} \frac{(\nu_m,\varphi_j)_{\mathcal{H}^*}(\nu_l,\varphi_j)_{\mathcal{H}^*}}{(1/2-\lambda_j)(\lambda(\omega)-\lambda_j)}.$$

- $(\nu_m, \varphi_0)_{\mathcal{H}^*} = 0$ ;  $\varphi_0$ : eigenfunction of  $\mathcal{K}_D^*$  associated to 1/2.
- Quasi-static plasmonic resonance: dist $(\lambda(\omega), \sigma(\mathcal{K}_D^*))$  minimal  $(\Re e \varepsilon_c(\omega) < 0)$ .

<sup>8</sup>with P. Millien, M. Ruiz, H. Zhang, Arch. Ration. Mecha Anal., 2017. Bio-inspired sensing and imaging Habib Ammari

• Enhancement of the scattering and absorption cross-sections  $Q^s$  and  $Q^a$  at plasmonic resonances<sup>9</sup>:

 $Q^{s} + Q^{s} (= \text{extinction cross-section } Q^{e}) \propto \Im m \operatorname{Trace}(M(\lambda(\omega), D));$  $Q^{s} \propto |\operatorname{Trace}(M(\lambda(\omega), D))|^{2}.$ 

<sup>9</sup>with P. Millien, M. Ruiz, H. Zhang, Arch. Ration. Mech. Anal., 2017. ≥ ∽ a Bio-inspired sensing and imaging Habib Ammari

• Single nanoparticle imaging<sup>10</sup>:

# $\max_{z^S} I(z^S, \omega)$

- $I(z^{s}, \omega)$ : imaging functional;  $z^{s}$ : search point.
- Resolution: limited only by the signal-to-noise-ratio.
- Cross-correlation techniques: robustness with respect to medium noise.



<sup>10</sup>with J. Garnier, P. Millien, SIAM J. Imag. Sci., 2014.

• Blow-up of the electric field in the gap at the plasmonic resonances<sup>11</sup>:

$$\nabla u = O(\frac{1}{(\delta/R)^{3/2} \ln(R/\delta)}).$$



Reconstruction from plasmonic spectroscopic data<sup>12</sup>. ٠



<sup>12</sup>with M. Ruiz, S. Yu, H. Zhang, SIAM J. Imag. Sci., Parts I & II, 2018. Habib Ammari

# Concluding remarks

- Resolution, stability, and specificity bio-inspired enhancement techniques:
  - Physics-based learning approach.
  - Multi-frequency imaging.
  - Differential imaging.
  - Nanoparticle imaging.
- Other applications: autonomous robotics
  - Equip autonomous robots with a "electric and acoustic sense perception".
  - Provide autonomous robots, by mimicking weakly electric fish and bats, with detection and classification capabilities in dark or turbid environments.
  - Complex targets; tracking of the position and orientation of mobile targets by extended Kalman filtering; autonomous navigation, ....