

## Discretization of generalized convection diffusion equations

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The project is focused on discretizations of generalized convection diffusion equations of discrete differential forms. These provide models both for the classical convection diffusion equation  $-\epsilon * \Delta u + \beta \nabla u + \gamma u = f$  as well as for the transport of magnetic fields in moving conducting fluids (magnetohydrodynamics).

We pursue both semi-Lagrangian and Eulerian techniques, the former for the transient problem, the latter also for the stationary case. The challenge is robustness and optimal a priori error estimates in the case of vanishing diffusion  $\epsilon \rightarrow 0$ .

In the project both semi-Lagrangian and DG-type Eulerian techniques could be extended to the generalized convection-diffusion problems for discrete differential forms. Exterior calculus involving the Lie derivative could be harnessed to achieve streamlined formulations. Sharper a priori convergence estimates beyond what was known before even for the scalar problem could be developed.

## Publications resulting from the project

- [1] H. HEUMANN, *Eulerian and Semi-Lagrangian Methods for Convection-Diffusion of Differential Forms*, ETH dissertation, ETH Zürich, Zürich, 2011. URL.
- [2] H. HEUMANN AND R. HIPTMAIR, *Extrusion contraction upwind schemes for convection-diffusion problems*, Report 2008-30, SAM, ETH Zürich, 2008.
- [3] —, *Eulerian and semi-lagrangian methods for convection-diffusion for differential forms*, Disc. Cont. Dyn. Sys., 29 (2010), pp. 1471–1495.
- [4] —, *Convergence of lowest order semi-lagrangian schemes*, Report 2011-47, SAM, ETH Zürich, 2011. Submitted to Found. Comp. Math.
- [5] —, *Refined convergence theory for semi-Lagrangian schemes for pure advection*, Report 2011-60, SAM, ETH Zürich, Zürich, Switzerland, 2011.
- [6] H. HEUMANN, R. HIPTMAIR, K. LI, AND J.-C. XU, *Semi-lagrangian methods for advection of differential forms*, Report 2011-21, SAM, ETH Zürich, Zürich, Switzerland, 2011. Submitted to BIT.
- [7] H. HEUMANN, R. HIPTMAIR, AND J. XU, *A semi-Lagrangian method for convection of differential forms*, Report 2009-09, SAM, ETH Zürich, Zürich, Switzerland, 2009.