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Plane Wave Discontinuous Galerkin Methods

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Standard low order Lagrangian finite element discretization of boundary value problems for the Helmholtz equation $-\Delta u - \omega^2 u = f$ are afflicted with the so-called pollution phenomenon: though for sufficiently small $h\omega$ (h being the meshwidth) an accurate approximation of u is possible, the Galerkin procedure fails to provide it. Attempts to remedy this have focused on incorporating extra information in the form of plane wave functions $\boldsymbol{x} \mapsto \exp(i\omega \boldsymbol{d} \cdot \boldsymbol{x})$, $|\boldsymbol{d}| = 1$, into the trial spaces.

This can easily done in the framework of discontinuous Galerkin (DG) method employing local trial spaces spanned by a few plane waves. In a sense, this is a special case of a Trefftz-type approximation. In this project a complete a priori discretization error of both the h- and p-version of general Trefftz DG methods for the Helmholtz equation and Maxwell's equations in the frequency domain was accomplished. Particular emphasis was put on elaborating the dependence of all constants on the frequency.

In addition, very detailed approximation estimates for plane waves and other common systems of Trefftz functions could be found. The main tool was Vekua's venerable theory, which allows to harness approximation estimates for harmonic functions. These estimates can also be applied to elastic waves.

Publications resulting from the project

[1] R. Hiptmair, A. Moiola, and I. Perugia, *Error analysis of Trefftz-discontinuous Galerkin methods for the time-harmonic Maxwell equations*, Math. Comp., (2011). Accepted.

[2] ——, *Plane wave discontinuous Galerkin methods for the 2d Helmholtz equation: Analysis of the p-version*, SIAM J. Numer. Anal., 49 (2011), pp. 264–284.

[3] ——, *Stability results for the time-harmonic maxwell equations with impedance boundary conditions*, Math. Model Meth. Appl. Sci., 21 (2011), pp. 2263–2287.

[4] A. Moiola, *Plane wave approximation in linear elasticity*, Report 2011-20, SAM, ETH Zürich, Zurich, Switzerland, 2011.

[5] A. Moiola, R. Hiptmair, and I. Perugia, *Plane wave approximation of homogeneous Helmholtz solutions*, ZAMP, 62 (2011), pp. 809–837. DOI: 10.1007/s00033-011-0147-y.

[6] ——, *Vekua theory for the Helmholtz operator*, ZAMP, 62 (2011), pp. 779–807. DOI: 10.1007/s00033-011-0142-3.