Proseminar in Autumn Semester 2008

Convolution Quadrature

Lecturers	:	Prof. R. Hiptmair
	:	Dr. Carlos Jerez-Hanckes

Venue	:	HG E 41
Time	:	Mo 16:15-18:00
First session	:	12.10.2009
Prep meeting	:	Mo 21.09.2008, 16:15, HG E 41
Contact	:	R. Hiptmair, hiptmair@sam.math.ethz.ch
	:	C. Jerez-Hanckes, carlos.jerez.hanckes@sam.math.ethz.ch
Prerequisites	:	Knowledge of calculus and numerical methods as acquired during the first two years of studies in mathematics, RW/CSE, and physics
Audience	:	BSc/MSc Students of Mathematics, Physics, RW/CSE, from the 3rd year

Description:

Convolution integrals like

$$(f * g)(t) = \int_0^t f(t-\tau)g(\tau)\,\mathrm{d}\tau\ , \quad t \geq 0\ ,$$

describe the response of causal linear time-invariant systems to an input signal g. The task is to evaluate $f * g$ at discrete times approximately, when only the Laplace transform $F(s)$ of $f(t)$ is known.

So-called *convolution quadrature methods* pioneered by C. Lubich serve this purpose and will be the focus of this seminar. They are derived using the z-transform, complex analysis techniques, and multi-step integration schemes for ordinary differential equations. This seminar will study the theoretical foundations of these methods, their numerical analysis, efficient implementation, and applications based on research publications.

Presentations:

The seminar will comprise up to 10 student presentations of a duration of about 60 minutes. They should be partly based on PDF slides prepared using the BEAMER LaTeXpackage (or LaTeXbased tools under MacOS). The presentations should be done using a laptop computer (which can be provided). Speakers are advised to elaborate technical manipulations and proofs on the blackboard. MATLAB demonstration of simple numerical experiments is expected whenever appropriate. The lecture slides in PDF format should be made available immediately after the presentation.

Available topics (tentative)

1. The continuous and discrete Laplace transform [BR84, Sect I.1-I.10] [Jur82, Sect I.1-I.5], Presentation slides by *Jan Ernest*
2. Numerical Laplace transform [Tal79, MR90, Wei06, DM79] [C.J.]
3. Multi-step integrators [DB02, Ch. 7], [HNW93, Sect III.1-II.4], [HW91, Sect. V.1] [R.H]
4. Convolution quadrature: Basics [Lub88a, Sect. 1], [Lub04, Sect. 1] [R.H.]
5. Convolution quadrature: Error analysis [Lub88a, Sect. 2-4], [Lub04, Sect. 2-3] [C.J.]
6. Convolution quadrature: Efficient implementation [Lub88b, Sect. 7], [Hen79] [R.H.]
7. Runge-Kutta convolution quadrature [LO93] [C.J.]
8. Application: Volterra integral equations [Lub85] and [Lub88b, Sect. 11] [C.J.]
9. Application: Absorbing boundary conditions [LS02] [R.H.]
10. Oblivious convolution quadrature [SLFL65] [R.H.]
11. Application: time-domain integral equations [AD+09, HKS09, SA97] [C.J.]

Dates for presentations:

Date	Speaker	Topic #
26.10.2009	Jan C. Ernest	1
2.11.2009	[R. Hiptmair]	3
9.11.2009	Alberto D.M. Paganini	4
30.11.2009	Alejandro Ojeda-Gonzalez	5
7.12.2009	Andrey Pethukov	10
14.12.2009	Albert M. Altarovici	8

References

[AAB+08] X. Antoine, A. Arnold, C. Besse, M. Ehrhardt, and A. Schädle. A review of transparent and artificial boundary conditions techniques for linear and nonlinear Schrödinger equations. *Comm. Comput. Phys.*, 4(4):729–796, 2008.

[AD+09] A. Aimi, , M. Diligenti, C. Guardasoni, I. Mazzieri, and S. Panizzi. An energy approach to space-time galerkin bem for wave propagation problems. *Int. J. Numer. Meth. Engr.*, 2009. Published Online: 19 Jun 2009.

[BR84] R. Bellman and R. Roth. *The Laplace Transform*. World Scientific Publishing, Singapore, 1984.

[DB02] P. Deuflhard and F. Bornemann. *Numerische Mathematik II*. DeGruyter, Berlin, 2 edition, 2002.

[DM79] B. Davies and B. Martin. Numerical inversion of the Laplace transform: A survey and comparison of methods. *J. Comp. Phys.*, 33(1):1–32, 1979.

[Hen79] P. Henrici. Fast fourier methods in computational complex analysis. *SIAM Rev.*, 21(4):481–527, 1979.

[HKS09] W. Hackbusch, W. Kress, and S. Sauter. Sparse convolution quadrature for time domain boundary integral formulations of the wave equation. *IMA J. Numer. Anal.*, 29(1):158–179, 2009.

[HLS85] E. Hairer, C. Lubich, and M. Schlichte. Fast numerical solution of non-linear Volterra convolution equations. *SIAM J. Sci. Stat. Comput.*, 6(3):532–541, 1985.

[HNW93] E. Hairer, S.P. Norsett, and G. Wanner. *Solving Ordinary Differential Equations I. Nonstiff Problems*. Springer-Verlag, Berlin, Heidelberg, New York, 2 edition, 1993.

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[Jur82] E. Jury. *Theory and Application of the z-Transform Method*. Robert E. Krieger Publishing Company, Malabar, 1982.

[LO93] Ch. Lubich and A. Ostermann. Runge-Kutta methods for parabolic equations and convolution quadrature. *Math. Comp.*, 60(201):105–131, 1993.

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[LS09] A.R. Laliena and F.-J. Sayas. Theoretical aspects of the application of convolution quadrature to scattering of acoustic waves. *Numer. Math.*, 112(4):637–678, 2009.

[Lub85] C. Lubich. Fractional linear multistep methods for Abel-Volterra integral equations of the second kind. *Math. Comp.*, 45(172):463–469, 1985.

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[Lub88b] C. Lubich. Convolution quadrature and discretized operational calculus. II. *Numer. Math.*, 52(4):413–425, 1988.

[Lub04] C. Lubich. Convolution quadrature revisited. *BIT*, 44(3):503–514, 2004.

[MR90] A. Murlio and M. Rizzardi. Talbot's method of the Laplace inversion problems. *ACM Transactions on Mathematical Software*, 16(2):158–168, 1990.

[SA97] M. Schanz and H. Antes. Application of operational quadrature methods in time domain boundary element methods. *Meccanica*, 32(3):179–186, 1997.

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[Wei06] J.A.C. Weideman. Optimizing Talbot's contours for the inversion of the Laplace transform. *SIAM J. Numer. Anal.*, 44(6):2342–2362, 2006.

The articles can be obtained from here.