Proseminar in Autumn Semester 2008

Numerical Methods for Oscillatory Integrals

Lecturer	: Prof. R. Hiptmair
Venue	: HG E 41
Time	: Mo 15:15-17:00
First session	: 13.10.2008
Prep meeting	: Mo 22.09.2008, 15:15, HG E 33.1
Contact	: R. Hiptmair, hiptmair@sam.math.ethz.ch
Prerequisites	: Basic knowledge of calculus as acquired during the first year of mathematics/physics curriculum. For some presentations skills in complex analysis are required. Facts about numerical quadrature as taught in the introductory numerical mathematics course.
Audience	: Students of Mathematics, Physics, CSE, and Computer Science from the 2nd year

Description:

By oscillatory integrals we mean integrals of the form

$$I_\omega(f) := \int_a^b f(x)\exp(i\omega g(x))\,\mathrm{d}x \tag{1}$$

and their multi-dimensional generalizations. Here, f and g are known functions, maybe even known as analytic expressions, and $\omega > 0$ is the frequency parameter.

We seek *efficient* methods for the highly accurate numerical approximation of (1) for large ω and fairly general f. This task is encountered in many contexts like numerical (Fourier) transformation, the solution of special ordinary differential equations, and in asymptotic integral equations methods for computational wave propagation. The challenge is that standard quadrature schemes become prohibitively expensive as they must capture the oscillations in the integrand of (1).

This seminar studies special methods, whose numerical costs do not increase as $\omega \to 0$ without sacrificing accuracy.

Presentations:

The seminar will comprise up to 10 student presentations of a duration between 45 and 65 minutes. They should be partly based on PDF slides prepared using the BEAMER LaTeXpackage (or LaTeXbased tools under MacOS). The presentations should be done using a laptop computer (which can be provided). Speakers are advised to elaborate technical manipulations and proofs on the blackboard. MATLAB demonstration of simple numerical experiments is expected whenever appropriate.

Available topics:

1. –: Asymptotic expansions for oscillatory integrals (method of stationary phase, Laplace method): Chapter 2, Sections 3.1–3.5 and 5.1–5.4 from [14]. See also [3, Sect. 1.2.2], [18, Sect. 2.2, 2.3].
2. **24.11.2008**: Collocation methods (Levin-type methods) (Marco Läubli): [11–13]
3. —: Expansion methods: [1, 2]
4. —: Filon-type methods I: [6] (except Sects. 2 & 5) and [7]
5. **1.12.2008**: Asymptotic and Filon-type methods (Timon Stucki): [8, 9]
6. —: Moment-free methods: [15, 17]
7. —: Multidimensional Levin-type methods: [16]
8. —: Method of steepest descent: Sections 4.1–4.5 from [14].
9. **8.12.2008**: Numerical steepest descent (Benny Löffel): [4]
10. —: Numerical steepest descent: the multidimensional case [5]

Additional introductory material [3, 10, 18, 19].

References

[1] ▷ G. Evans, *Two robust methods for irregular oscillatory integrals over a finite range*, Appl. Num. Math., 14 (1994), pp. 383–395.

[2] ▷ ——, *An expansion method for irregular oscillatory integrals*, Int. J. Computer Mathematics, 63 (1997), pp. 137–148.

[3] ▷ D. Huybrechs and S. Olver, *Highly oscillatory quadrature*, Proceedings of HOP INI Program, (2008).

[4] ▷ D. Huybrechs and S. Vandewalle, *On the evaluation of highly oscillatory integrals by analytic continuation*, SIAM J. Numer. Anal., 44 (2006), pp. 1026–1048.

[5] ▷ ——, *The construction of cubature rules for multivariate highly oscillatory integrals*, Math. Comp., 76 (2007), pp. 1955–1980.

[6] ▷ A. Iserles, *On the numerical quadrature of highly oscillatory integrals I: Fourier transforms*, IMA J. Numer. Anal., 24 (2004), pp. 365–391.

[7] ▷ ——, *On the numerical quadrature of highly oscillatory integrals II: Irregular oscillators*, IMA J. Numer. Anal., 25 (2005), pp. 25–44.

[8] ▷ A. Iserles and S. Nørsett, *On quadrature methods for highly oscillatory integrals and their implementation*, BIT, 44 (2004), pp. 755–772.

[9] ▷ A. Iserles and S. Nørsett, *Efficient quadrature of highly oscillatory integrals using derivatives*, Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci., 461 (2005), pp. 1383–1399.

[10] ▷ A. Iserles, S. Nørsett, and S. Olver, *Highly oscillatory quadrature: The story so far*, in Proceedings of EnuMath, Santiago de Compostela, A. Bermudez de Castro, ed., Berlin, 2006, Springer, pp. 97–118.

[11] ▷ D. Levin, *Procedures for computing one- and two-dimensional integrals of functions with rapid irregular oscillations*, Math. Comp., 38 (1982), pp. 531–538.

[12] ▷ ——, *Fast integration of rapidly oscillatory functions*, J. Comp. Appl. Math., 67 (1996), pp. 95–101.

[13] ▷ ——, *Analysis of a collocation method for integrating rapidly oscillatory functions*, J. Comp. Appl. Math., 78 (1997), pp. 131–138.

[14] ▷ P. Miller, *Applied Asymptotic Analysis*, vol. 75 of Graduate Studies in Mathematics, AMS, Providence, RI, 2006.

[15] ▷ S. Olver, *Moment-free numerical integration of highly oscillatory functions*, IMA J. Numer. Anal., 26 (2006), pp. 213–227.

[16] ▷ ——, *On the quadrature of multivariate highly oscillatory integrals over non-polytope domains*, Numer. Math., 103 (2006), pp. 643–665.

[17] ▷ ——, *Moment-free numerical approximation of highly oscillatory integrals with stationary points*, European J. Appl. Math., 18 (2007), pp. 435–447.

[18] ▷ ——, *Numerical Approximation of Highly Oscillatory Integrals*, phd thesis, University of Cambridge, Cambridge, UK, 2008.

[19] ▷ S. Vandewalle, *Numerical integration of highly oscillatory functions based on analytic continuation*. Lecture Slides, HOP Workshop, Newton Insitutute Cambridge, 2007.