STABLE LÉVY MOTION APPROXIMATION IN COLLECTIVE RISK THEORY

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Abstract

Collective risk theory is concerned with random fluctuations of the total net assets, the risk reserve, of an insurance company. In this paper we consider weak approximations in risk theory which are especially relevant whenever the claim experience allows for heavy-tailed claims. We approximate the risk process by an α-stable Lévy motion ($1 < \alpha < 2$) with drift. The ruin probability within a finite time horizon is estimated. Finally, a numerical example is presented.

Keywords: Risk theory, Ruin probability, Heavy-tailed claims, α-stable Lévy motion, Weak convergence in $D[0,\infty)$.

1 Introduction

Consider a company which only writes ordinary insurance policies such as accident, disability, health and whole life. The policyholders pay premiums regularly and at certain random times make claims to the company. A policyholder's premium, the gross risk premium, is a positive amount composed of two components. The net risk premium is the component calculated to cover the payments of claims on the average, while the security risk premium, or safety loading, is the component which protects the company from large deviations of claims from the average and also allows an accumulation of capital. When a claim occurs the company pays the policyholder a positive amount called the positive risk sum.

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As a mathematical model for this situation we shall assume that claims occur at jumps of a point process $N = (N(t) : t \geq 0)$. While most work in collective risk theory assumes that N is a Poisson process, this restrictive assumption plays no role in our analysis. The successive claims $(Y_k : k \in \mathbb{N})$ are assumed to form a sequence of independent, identically distributed (iid) random variables with $E[Y_k] = \mu > 0$. Furthermore, we shall assume that the initial risk reserve of the company is $u > 0$ and that the policyholders pay a gross risk premium of $c > 0$ per unit time. So the risk process has the form

$$R(t) = u + ct - \sum_{k=1}^{N(t)} Y_k \ . \tag{1}$$

Also we define the ruin time T as the first time the company has a negative risk reserve, i.e.

$$T = \inf\{t > 0 : R(t) < 0\}$$

if the set is nonempty and $T = \infty$ otherwise. In order to avoid $T < \infty$ a.s. we assume that the net profit condition $\lim_{t\to\infty} E[R(t)]/t > 0$ holds. One of the key problems of collective risk theory concerns calculating the ruin probability $\Psi(u) = P\{T < \infty \mid R(0) = u\}$, i.e. the probability that the risk process ever becomes negative. On the other hand, an insurance company will typically be interested in the probability that ruin occurs before time t. The finite-time ruin probability $\Psi(u, t)$ is defined by $\Psi(u,t) = P\{T \leq t \mid R(0) = u\}$. Many of the results for these distributions are complicated expressions which have been obtained using analytical methods. For a comprehensive treatment of the theory up to 1955 the reader should consult Cramér (1955). A more recent account of the theory is available in Chapter 7 of Takács (1967), Grandell (1991) and Schmidli (1992), where further references are to be found.

Already in 1940 Hadwiger compared a discrete-time risk process with diffusion. This can be viewed, though theoretically not comparable with the modern approach, as the first treatment of diffusion approximations in risk theory. A more modern version, based on weak convergence, is due to Iglehart (1969). The idea is to let the number of claims grow in a unit time interval and to make the claim sizes smaller in such a way that the risk process converges weakly to a diffusion. In this paper we shall consider weak approximations where the idea is to approximate the risk process with an α-stable Lévy motion with drift. While the classical theory of diffusion approximation requires short-tailed claims, this assumption can be dropped in our approach, hence allowing for extremal events in the sense of Embrechts and Schmidli (1994).

The paper is organized as follows. In section 2, we first recall the definition and some basic properties of stable distributions and stable processes. We then define a sequence of risk processes and show that they converge weakly to an α-stable Lévy motion with drift. We illustrate the convergence when the claim arrival process is an arbitrary renewal process. Section 3 deals with the convergence of functionals of the risk process. It is shown that the finite-time passage probabilities converge. In section 4 we establish upper bounds for the probability of the first 0-down-crossing of our α-stable Lévy motion. Finally, in section 5 we present a numerical example.

As already stated above, our approximations are especially relevant whenever the claim experience allows for heavy-tailed claims. Discussions of the relevance of heavy-tailed conditions can for instance be found in Embrechts and Veraverbeke (1982) or Embrechts et al. (1996). Besides the intrinsic importance of our results in the field of large claims, our paper also provides the necessary background for further research in the area of insurance and finance whenever 'standard' light-tailed conditions are violated.

2 Weak Convergence of Risk Reserve Processes

Let $D = D[0, \infty)$ be the space of all real-valued right continuous functions on $[0, \infty)$ with limits from the left, endowed with the Skorokhod topology. Then D is a complete and separable metric space, see for instance Lindvall (1973). All stochastic processes in this paper are assumed to be in D.

Definition 1 *A sequence $(X^{(n)} : n \in \mathbb{N})$ of stochastic processes is said to converge weakly in the Skorokhod topology to a stochastic process X if for every bounded continuous functional f on D it follows that*

$$\lim_{n\to\infty} E[f(X^{(n)})] = E[f(X)] \ .$$

In this case one writes $X^{(n)} \Rightarrow X$.

Gaussian distributions and processes have long since been studied and their utility in stochastic modeling is well-accepted. However, analysis of financial and insurance data often uncovers oscillations that indicate the presence of heavy tails. Stable processes are an appropriate source of heavy tailed probabilistic models.

Definition 2 *A stochastic process $Z_\alpha = (Z_\alpha(t) : t \geq 0)$ is called (standard) α-stable Lévy motion if*

(i) $Z_\alpha(0) = 0$ *a.s.*

(ii) Z_α *has independent increments.*

(iii) $Z_\alpha(t) - Z_\alpha(s) \sim S_\alpha((t-s)^{1/\alpha}, \beta, 0)$ *for any* $0 \le s < t < \infty$ *and for some* $0 < \alpha \le 2,\ |\beta| \le 1$.

See Figure 1 for some simulated sample paths in the case $\alpha = 1.2$, $\beta = 0$. We use

Figure 1: Simulation of 10 sample paths of α-stable Lévy motion with $\alpha = 1.2$, $\beta = 0$.

the notation $X \sim S_\alpha(\sigma, \beta, \mu)$ to indicate that the random variable X has a stable distribution, characterized by the four parameters $0 < \alpha \le 2$ (index of stability), $\sigma > 0$ (scale parameter), $-1 \le \beta \le 1$ (skewness parameter) and $\mu \in \mathbb{R}$ (shift parameter). The definition of an univariate stable distribution derives from the "stability" property that the family of stable distributions is preserved under convolution. More precisely, a random variable X is said to have a stable distribution if for any positive numbers a and b, there is a positive number c and a real number d such that

$$aX' + bX'' \stackrel{d}{=} cX + d\,,$$

where X' and X'' are independent copies of X and "$\stackrel{d}{=}$" means equality in law. Explicit forms of stable density functions only exist in the cases $\alpha = 1/2$ (Lévy distribution), $\alpha = 1$ (Cauchy distribution) and $\alpha = 2$ (Normal distribution). The tails of non-Gaussian stable distributions decrease like a power function. The rate of decay mainly depends on the parameter α. The smaller α, the slower the decay and the heavier the tails. For a stable random variable X with index $\alpha < 2$ one has $E[|X|^\delta] = \infty$ for any $\delta \ge \alpha$ and $E[|X|^\delta] < \infty$ for $0 < \delta < \alpha$.

When $\alpha = 2$, the process Z_α is Brownian motion. The α-stable Lévy motions are $1/\alpha$-self-similar (unless $\alpha = 1,\ \beta \neq 0$), i.e. for all $c > 0$, $(Z_\alpha(ct) : t \ge 0)$ and

$(c^{1/\alpha} Z_\alpha(t) : t \geq 0)$ have the same finite-dimensional distributions. Since Z_α has stationary and independent increments, the characteristic function of $Z_\alpha(t)$ allows the representation

$$E[e^{i\theta Z_\alpha(t)}] = e^{t\xi(\theta)} ,$$

where $\theta \in \mathbb{R}$ and

$$\xi(\theta) = \int_{\mathbb{R}} \left(e^{i\theta y} - 1 - \frac{i\theta y}{1+y^2} \right) \Pi(dy)$$

with

$$\Pi(dy) = \frac{p}{y^{1+\alpha}} 1_{(0,\infty)}(y)\, dy + \frac{q}{y^{1+\alpha}} 1_{(-\infty,0)}(y)\, dy, \quad p+q>0 .$$

One sets $\beta = (p-q)/(p+q)$. If for instance $\beta = -1$, i.e. $p=0$, the Lévy measure Π attributes no mass to the positive half line and consequently there are no positive jumps of Z_α. For a comprehensive survey of properties of α-stable random variables and α-stable Lévy motions we refer to Janicki and Weron (1994) or Samorodnitsky and Taqqu (1994).

The sequence $(Q^{(n)} : n \in \mathbb{N})$ of risk processes is given as follows. For every $n \in \mathbb{N}$ let $u^{(n)} > 0$ denote the initial risk reserve, $c^{(n)} > 0$ the premium rate and $N^{(n)}$ the corresponding point process of claim arrivals. The claim sizes are denoted by $(Y_k^{(n)} : k \in \mathbb{N})$. So

$$Q^{(n)}(t) = u^{(n)} + c^{(n)}t - \sum_{k=1}^{N^{(n)}(t)} Y_k^{(n)} , \qquad t \geq 0. \tag{2}$$

We now assume that the claims are of the form $Y_k^{(n)} = Y_k/\varphi(n)$, where $(Y_k : k \in \mathbb{N})$ is a sequence of iid random variables with common distribution function F and mean μ such that

$$\frac{1}{\varphi(n)} \sum_{k=1}^{n} (Y_k - \mu) \Rightarrow Z_\alpha(1) , \quad n \to \infty . \tag{3}$$

The function φ is given by $\varphi(n) = n^{1/\alpha}L(n)$, where L is slowly varying at infinity. In other words, F is in the domain of attraction of $Z_\alpha(1)$. Because $\alpha = 2$ leads to the well-known Brownian diffusion approximation as treated for instance in Schmidli (1994), we shall restrict ourselves from now on to the case $1 < \alpha < 2$. This means that the random variables Y_k do not have finite second moments and their distribution possesses long-tail behaviour. For further properties of the above mentioned conditions see for instance Bingham et al. (1987). If not stated otherwise explicitly, we make for the rest of this paper the following assumption:

Assumption 1 *The index of stability is*

$$1 < \alpha < 2.$$

The condition $\alpha > 1$ is needed to guarantee a finite mean of the variable $Z_\alpha(t)$. We now define a process $Q = (Q(t) : t \geq 0)$ by

$$Q(t) = u + ct - \lambda^{1/\alpha} Z_\alpha(t) , \tag{4}$$

where u and c are positive numbers and $(Z_\alpha(t) : t \geq 0)$ is α-stable Lévy motion. Here λ is some positive constant which will be specified in the following theorem. Observe that the process in (4) for $\alpha < 2$ contrary to the Brownian case $(\alpha = 2)$ exhibits jumps. For some simulated sample paths of Q, see Figure 2 below. We now

Figure 2: Simulations of the process Q with $u = 20$, $c = 5$, $\lambda = 1$, $\alpha = 1.2$, $\beta = 1$.

show that, under certain conditions, the sequence $(Q^{(n)} : n \in \mathbb{N})$ converges weakly to the process Q.

Theorem 1 *Let the sequence* $(Y_k : k \in \mathbb{N})$ *be as above and let* $(N^{(n)} : n \in \mathbb{N})$ *be a sequence of point processes such that*

$$\frac{N^{(n)}(t) - \lambda n t}{\varphi(n)} \to 0 , \quad n \to \infty \tag{5}$$

in probability in the Skorokhod topology for some positive constant λ. *Assume also that*

$$\lim_{n\to\infty} \left(c^{(n)} - \lambda n \frac{\mu}{\varphi(n)} \right) = c , \qquad \lim_{n\to\infty} u^{(n)} = u .$$

Then

$$u^{(n)} + c^{(n)} t - \frac{1}{\varphi(n)} \sum_{k=1}^{N^{(n)}(t)} Y_k \Rightarrow u + ct - \lambda^{1/\alpha} Z_\alpha(t) , \quad n \to \infty \tag{6}$$

in the Skorokhod topology.

In order to prove Theorem 1 we will need the following theorem on random time transformation, see Whitt (1980), where it is shown that the composition mapping is continuous.

Proposition 1 *Let* $(Z_n : n \in \mathbb{N})$ *,* Z *be processes in* D *and suppose that* $Z_n \Rightarrow Z$ *. Let* $(N_n : n \in \mathbb{N})$ *be a sequence of processes with nondecreasing sample paths starting from 0 such that* $N_n \Rightarrow \lambda I$ *, where* $\lambda > 0$ *and* I *denotes the identity map on* $[0, \infty)$ *. For each* $n \in \mathbb{N}$ *,* Z_n *and* N_n *are assumed to be defined on the same probability space. Then*

$$Z_n(N_n) \Rightarrow Z(\lambda I) \ , \quad n \to \infty \ . \tag{7}$$

Proof of Theorem 1. Let us write $Q^{(n)}(t)$ in the following form

$$\begin{aligned} Q^{(n)}(t) &= u^{(n)} + c^{(n)}t - \frac{1}{\varphi(n)} \sum_{k=1}^{N^{(n)}(t)} Y_k \\ &= u^{(n)} + t\left(c^{(n)} - \lambda n \frac{\mu}{\varphi(n)}\right) - \mu\left(\frac{N^{(n)}(t) - \lambda n t}{\varphi(n)}\right) - \frac{1}{\varphi(n)} \sum_{k=1}^{N^{(n)}(t)} (Y_k - \mu) \ . \end{aligned}$$

From assumption (5) we obtain

$$\mu\left(\frac{N^{(n)}(t) - \lambda n t}{\varphi(n)}\right) \to 0 \ , \quad n \to \infty$$

in probability in the Skorokhod topology. It follows from assumption (3) that

$$\frac{1}{\varphi(n)} \sum_{k=1}^{[nt]} (Y_k - \mu) \Rightarrow Z_\alpha(t) \ , \quad n \to \infty \tag{8}$$

in the Skorokhod topology (see Skorokhod (1957)). From (8) and (5) and Proposition 1 we obtain that

$$\frac{1}{\varphi(n)} \sum_{k=1}^{N^{(n)}(t)} (Y_k - \mu) \Rightarrow \lambda^{1/\alpha} Z_\alpha(t) \ , \quad n \to \infty$$

in the Skorokhod topology. Because

$$u^{(n)} + t\left(c^{(n)} - \lambda n \frac{\mu}{\varphi(n)}\right) - \mu\left(\frac{N^{(n)}(t) - \lambda n t}{\varphi(n)}\right)$$

converges to $u + ct$ in probability in the Skorokhod topology, the proof is complete.

$\square$

The previous theorem shows that the distribution of a risk process in the long-tailed case can be approximated by the distribution of a process of the type (4). In the next section we investigate the question of the convergence of the associated ruin probabilities . In section 5, we shall look more closely at the numerical properties of the approximation derived in Theorem 1.

REMARK. Note that the point process N need not be a Poisson process in order to fulfil (5). Indeed, condition (5) holds for a wider class of point processes. For instance, let N be an arbitrary renewal process:

$$N(t) = \max\{n : \sum_{k=1}^{n} T_k \leq t\} ,$$

where the inter-occurrence times $(T_k : k \in \mathbb{N})$ are assumed to be independent positive random variables. We define

$$N^{(n)}(t) = N(nt) . \tag{9}$$

Let $B = (B(t) : t \geq 0)$ denote standard one-dimensional Brownian motion. Then the following proposition holds.

Proposition 2 *Let $(N(t) : t \geq 0)$ be a renewal process with inter-occurrence times $(T_k : k \in \mathbb{N})$ and assume that there exist a positive constant λ and a slowly varying function L such that*

$$\frac{1}{\varphi(n)} \sum_{k=1}^{[nt]} \left(T_k - \frac{1}{\lambda}\right) \Rightarrow B(t) , \quad n \to \infty \tag{10}$$

in the Skorokhod topology where $\varphi(n) = \sqrt{n}L(n)$. Then, for $1 < \alpha < 2$, we have

$$\frac{N(nt) - \lambda nt}{n^{1/\alpha}} \to 0 , \quad n \to \infty \tag{11}$$

in probability in the Skorokhod topology.

Proof. It suffices to check that

$$P\left\{\sup_{0 \leq s \leq t} \frac{|N(ns) - \lambda ns|}{n^{1/\alpha}} > \epsilon\right\} \to 0 , \quad n \to \infty .$$

If

$$\sup_{0 \leq s \leq t} \frac{|N(ns) - \lambda ns|}{n^{1/\alpha}} > \epsilon ,$$

then there exists $s \in [0, t]$ with $N(ns) - \lambda ns > \epsilon n^{1/\alpha}$ or $N(ns) - \lambda ns < -\epsilon n^{1/\alpha}$. In the first case we obtain

$$\sum_{k=1}^{[\lambda ns + \epsilon n^{1/\alpha}]} T_k < ns .$$

Define u such that $nu = \lambda ns + \epsilon n^{1/\alpha}$, then

$$\sum_{k=1}^{[nu]} \left(T_k - \frac{1}{\lambda} \right) < -\frac{\epsilon n^{1/\alpha}}{\lambda} \ .$$

Therefore

$$\sup_{0 \le u \le \lambda t + \epsilon n^{1/\alpha - 1}} \frac{1}{n^{1/\alpha}} \left| \sum_{k=1}^{[nu]} \left(T_k - \frac{1}{\lambda} \right) \right| > \frac{\epsilon}{\lambda} \ .$$

Note that if $1 < \alpha < 2$, then the above supremum is taken on a bounded interval. Because $\alpha < 2$ and $L(n)n^{-\delta}$ converges to 0 for every $\delta > 0$, assumption (10) implies that

$$\frac{1}{n^{1/\alpha}} \sum_{k=1}^{[nu]} \left(T_k - \frac{1}{\lambda} \right) \to 0 \ , \quad n \to \infty \tag{12}$$

in probability in the Skorokhod topology. If $N(ns) - \lambda ns < -\epsilon n^{1/\alpha}$ we obtain

$$\sum_{k=1}^{[\lambda ns - \epsilon n^{1/\alpha}]} T_k > ns \ .$$

For $nu = \lambda ns - \epsilon n^{1/\alpha}$ we have

$$\sum_{k=1}^{[nu]} \left(T_k - \frac{1}{\lambda} \right) > \frac{\epsilon n^{1/\alpha}}{\lambda} \ .$$

Thus

$$\sup_{0 \le u \le \lambda t - \epsilon n^{1/\alpha - 1}} \frac{1}{n^{1/\alpha}} \left| \sum_{k=1}^{[nu]} \left(T_k - \frac{1}{\lambda} \right) \right| > \frac{\epsilon}{\lambda} \ .$$

Finally from relation (12) we have

$$P \left\{ \sup_{0 \le s \le t} \frac{|N(ns) - \lambda ns|}{n^{1/\alpha}} > \epsilon \right\} \to 0 \ , \quad n \to \infty$$

and the proof is complete. $\square$

Note that (10) is true in the ordinary renewal theory situation where $(T_k : k \in \mathbb{N})$ is a sequence of iid random variables with mean $1/\lambda$ and variance σ^2, hence $\varphi(n) = \sqrt{n}\sigma$. In this case we have (see Billingsley (1968) p. 148)

$$\frac{N(nt) - \lambda nt}{\sqrt{n}} \Rightarrow \sigma \lambda^{3/2} B(t) \ , \quad n \to \infty \ . \tag{13}$$

3 The Convergence of Functionals of Risk Processes

Collective risk theory has paid considerable attention to the ruin functional and the associated ruin times $T^{(n)} = T(Q^{(n)})$ where

$$T(x) = \inf\{t > 0 \ : x(t) < 0\} \tag{14}$$

if the set $\inf\{t > 0 \ : x(t) < 0\}$ is not empty and $+\infty$ otherwise. In order to derive the result in Theorem 2 which permits us to compare the finite-time ruin probability by the 'ruin probability' of the corresponding weak approximation, we need the following technical lemma. Let $X = (X(t) : t \geq 0)$ be a càdlàg process. We define the process $\Delta X = (\Delta X(t) : t \geq 0)$ by $\Delta X(t) = X(t) - X(t-)$. Then

$$\{\Delta X \neq 0\} = \cup_{n=1}^{\infty}\{(\omega, t)\,;\, t \geq 0\,, t = T_n(\omega)\}\ ,$$

where $(T_n : n \in \mathbb{N})$ are stopping times, see Jacod and Shiryaev (1987), Proposition 1.32, p. 8.

Lemma 1 *Let* $Q = (Q(t) : t \geq 0)$ *be the process given in* (4) *and let* (τ_n) *be the sequence of all jump times of* Q. *Then*

$$P\{\cup_{n=1}^{\infty}\{Q(\tau_n -) = 0\}\} = 0\ . \tag{15}$$

Proof. It is sufficient to prove the property on a finite time interval. We use the series representation of stable processes to gain insight into the structure of α-stable Lévy motion. If Z_α denotes strictly α-stable Lévy motion with index $1 < \alpha < 2$ we can write $Z_\alpha(t)$ in the following form (see Samorodnitsky and Taqqu (1994) p. 151)

$$Z_\alpha(t) = C_\alpha^{1/\alpha} \sum_{i=1}^{\infty} (\gamma_i \Gamma_i^{-1/\alpha} 1(\tau_i \leq t) - \beta t b_i^{(\alpha)})\ ,$$

where $0 \leq t \leq 1$ and $(\gamma_i : i \in \mathbb{N})$ is a sequence of iid random variables satisfying

$$P\{\gamma_i = 1\} = 1 - P\{\gamma_i = -1\} = \frac{1+\beta}{2}\ .$$

Here $(\Gamma_i : i \in \mathbb{N})$ is a sequence of arrival times of a Poisson process with unit arrival rate and $(\tau_i : i \in \mathbb{N})$ is a sequence of iid random variables uniformly distributed on $[0,1]$. These three sequences are independent. Finally, C_α and $b_i^{(\alpha)}$ are some constants. The jump times (τ_i) are distributed uniformly over $[0,1]$, the direction of a jump is governed by (γ_i) and the height of the jumps, viewed in decreasing order, is distributed as the $-1/\alpha$ power of the arrival times of a Poisson process with unit intensity. In particular, $\Gamma_1^{-1/\alpha}$ is the height of the highest jump, $\Gamma_2^{-1/\alpha}$ is the height of the second highest jump and so on. Our process Q can then be written in the following form:

$$Q(t) = u + ct - \lambda^{1/\alpha} C_\alpha^{1/\alpha} \sum_{i=1}^{\infty} (\gamma_i \Gamma_i^{-1/\alpha} 1(\tau_i \le t) - \beta t b_i^{(\alpha)}) \ .$$

We prove that $P\{Q(\tau_k -) = 0\} = 0$. We have

$$\begin{aligned} & P\{Q(\tau_k-) = 0\} \\ &= P\{\Delta Q(\tau_k) = Q(\tau_k)\} \\ &= P\Big\{u + c\tau_k + \lambda^{1/\alpha} C_\alpha^{1/\alpha} \beta \tau_k b_k^{(\alpha)} - \lambda^{1/\alpha} C_\alpha^{1/\alpha} \sum_{i=1, i\neq k}^{\infty} (\gamma_i \Gamma_i^{-1/\alpha} 1(\tau_i \le \tau_k) - \beta \tau_k b_i^{(\alpha)}) = 0\Big\} \ . \end{aligned}$$

Using conditional probability and independence we obtain

$$\begin{aligned} & P\{Q(\tau_k-) = 0\} \\ &= \int_0^1 P\Big\{u + c\tau_k + \lambda^{1/\alpha} C_\alpha^{1/\alpha} \beta \tau_k b_k^{(\alpha)} \\ &\qquad -\lambda^{1/\alpha} C_\alpha^{1/\alpha} \sum_{i=1, i\neq k}^{\infty} (\gamma_i \Gamma_i^{-1/\alpha} 1(\tau_i \le \tau_k) - \beta \tau_k b_i^{(\alpha)}) = 0 \mid \tau_k = t\Big\}\, dt \\ &= \int_0^1 P\Big\{u + ct + \lambda^{1/\alpha} C_\alpha^{1/\alpha} \beta t b_k^{(\alpha)} \\ &\qquad -\lambda^{1/\alpha} C_\alpha^{1/\alpha} \sum_{i=1, i\neq k}^{\infty} (\gamma_i \Gamma_i^{-1/\alpha} 1(\tau_i \le t) - \beta t b_i^{(\alpha)}) = 0\Big\}\, dt \\ &= 0 \ . \end{aligned}$$

In the last equality we used

$$\begin{aligned} & P\Big\{u + ct + \lambda^{1/\alpha} C_\alpha^{1/\alpha} \beta t b_k^{(\alpha)} - \lambda^{1/\alpha} C_\alpha^{1/\alpha} \sum_{i=1, i\neq k}^{\infty} (\gamma_i \Gamma_i^{-1/\alpha} 1(\tau_i \le t) - \beta t b_i^{(\alpha)}) = 0\Big\} \\ &= P\Big\{u + ct - \lambda^{1/\alpha} Z_\alpha(t) = 0 \mid \tau_k > t\Big\} \\ &= 0 \end{aligned}$$

for all $0 < t < 1$. This concludes the proof. □

Now we are able to state and prove the following basic limit theorem.

Theorem 2 *Let T be the ruin functional defined in* (14). *If $Q^{(n)} \Rightarrow Q$ and Q is given in equation* (4), *then*

$$T(Q^{(n)}) \Rightarrow T(Q) \ . \tag{16}$$

Proof. Let x_n and x be càdlàg functions such that $x_n \to x$ in the Skorokhod topology. Assume that $T(x_n)$ does not converge to $T(x)$. Then we can find a

subsequence $T(x_n)$ which will tend to $t_0 \neq T(x)$. Let us first assume that $t_0 > T(x)$ and $T(x) < \infty$, we also allow for $t_0 = \infty$. We have $x(T(x)) \leq 0$. We can find $\delta > 0$ such that $T(x)+\delta < t_0$ and $x(T(x)+\delta) = x([T(x)+\delta]-)$ and $\epsilon < -x(T(x)+\delta)$ for some $\epsilon > 0$. Then for sufficiently large n we obtain $x_n(T(x)+\delta) < \epsilon + x(T(x)+\delta)$. Hence, $x_n(T(x)+\delta) < 0$ for n large enough.
Now let $t_0 < T(x)$, we also allow for $T(x) = \infty$, then $x_n(T(x_n)) \leq 0$ and $x_n(T(x_n))$ tends to $x(t_0)$ or $x(t_0-)$ (more precisely, a subsequence of $x_n(T(x_n))$ has that property). Thus $x(t_0) \leq 0$ or $x(t_0-) \leq 0$. In the case of $x(t_0) \leq 0$ we use the strong Markov property and the fact that $Z_\alpha(t) - ct$ crosses 0 infinitely many times in every right neighbourhood of $t = 0$. This follows from the law of iterated logarithm for stable processes (see Fristedt (1974) p. 361 and Mijnheer (1975) p. 45). In the case of $x(t_0-) \leq 0$, using Lemma 1, we have $x(t_0-) < 0$ for almost all trajectories. So we can find $\delta > 0$ such that $x(t_0 - \delta) < 0$ which is a contradiction.
We have shown that the functional T is almost surely continuous. The assertion now follows from the continuous mapping theorem, see Billingsley (1968) p. 31.

□

REMARKS.

1. It is easy to show that $P\{T(Q) = t\} = 0$. Consequently, Theorem 2 shows that the finite-time passage probabilities converge:

$$\lim_{n\to\infty} P\{T(Q^{(n)}) \leq t\} = P\{T(Q) \leq t\}$$

or equivalently

$$\lim_{n\to\infty} P\{\inf_{0\leq s\leq t} Q^{(n)}(s) < 0\} = P\{\inf_{0\leq s\leq t} Q(s) < 0\} .$$

Hence we can approximate the finite-time ruin probabilities by the probability of the first 0–downcrossing of the process (4).

2. In the Brownian case $(\alpha = 2)$ some additional conditions are required to show that the infinite-time ruin probabilities converge. We then have

$$\lim_{n\to\infty} P\{\inf_{s\geq 0} Q^{(n)}(s) < 0\} = P\{\inf_{s\geq 0} Q(s) < 0\}$$

The last expression is shown to be $\exp\{-\gamma u\}$ for some constant γ, see for instance Asmussen (1987) p. 265. In Furrer (1997) it is shown that for $1 < \alpha < 2$ and $\beta = 1$ we have $P\{\inf_{s\geq 0} Q(s) < 0\} = E_{\tilde\alpha}(-\kappa u^{\tilde\alpha})$, where $\tilde\alpha = \alpha - 1$, κ is some positive constant and $E_\sigma(x) = \sum_{n=0}^{\infty} x^n/\Gamma(1+\sigma n)$ denotes the

Mittag–Leffler function, a generalization of the exponential to which it reduces when $\alpha = 2$. Driftfree stable processes and functionals related to them have been studied extensively in the literature, see for instance Bingham (1973) and references therein. The distribution of the infimum of stable Lévy motions ($\beta = -1$) with a drift term is to our knowledge included only in Furrer (1997). The problem of the convergence of the infinite-time ruin probabilities for $1 < \alpha < 2$ which is less important for applications will be discussed in a forthcoming note.

4 The Ruin Probability within Finite Time

In the previous sections we showed that the process Q defined in (4) can be considered as an approximation for a risk process. Our aim in this section now is to investigate $P\{T(Q) \le t\}$, i.e. the probability that the process Q reaches the level 0 before time t. Whereas in the Brownian case $(\alpha = 2)$ the above probability can be calculated explicitly (see Asmussen (1987), p. 263), the case $1 < \alpha < 2$ turns out to be more delicate. We first consider the asymptotic behaviour of $P\{T(Q) \le t\}$ as the initial risk reserve u becomes large. If f and g are two functions, we mean by $f(x) \sim g(x)$ that $\lim_{x\to\infty} f(x)/g(x) = 1$.

Proposition 3 *Let Z_α be α-stable Lévy motion with skewness parameter $-1 < \beta \le 1$. Then*

$$P\{T(u + cs - \lambda^{1/\alpha} Z_\alpha(s)) \le t\} \sim C_\alpha \, \frac{1+\beta}{2} \, \lambda \; t \; (u + ct)^{-\alpha} \; , \quad u \to \infty \; , \tag{17}$$

where $C_\alpha = (1-\alpha)/(\Gamma(2-\alpha)\cos(\pi\alpha/2))$.

Proof. Since the distribution of $Z_\alpha(t)$ has a regularly varying right tail for $-1 < \beta \le 1$, we conclude that $P\{T(u + cs - \lambda^{1/\alpha} Z_\alpha(s)) \le t\} \sim P\{ct - \lambda^{1/\alpha} Z_\alpha(t) < -u\}$, $u \to \infty$ (see Willekens (1987)). The statement of the Theorem now follows using the tail behaviour of stable distributions, see Samorodnitsky and Taqqu (1994) p. 16. □

We shall next derive upper bounds for $P\{T(Q) \le t\}$, where the stable component is assumed to be symmetric. To this end we will use the Bochner theorem which says that symmetric α-stable Lévy motion can be obtained from Brownian motion by a random time change. More precisely, let $(X(t) : t \ge 0)$ be totally skewed $\alpha/2$-stable Lévy motion, i.e. $X(t) \sim S_{\alpha/2}(t^{2/\alpha}, 1, 0)$ and B standard Brownian motion,

$B(1) \sim N(0,1) = S_2(1/\sqrt{2},0,0)$ and assume that both processes are independent and defined on the same probability space. Then (see Janicki and Weron (1994) p. 33)

$$B(X(t)) \stackrel{d}{=} Z_\alpha(t) \tag{18}$$

and $B(X(t))$ is symmetric α-stable Lévy motion such that

$$B(X(t)) \sim S_\alpha(t^{1/\alpha}/(\sqrt{2}(\cos\frac{\pi\alpha}{4})^{1/\alpha}),0,0) \ .$$

Now let $S = (S(t) : t \geq 0)$ denote the maximum process of B, i.e. $S(t) = \sup_{0\leq s\leq t} B(s)$. Since B is continuous, $S(t) = \sup_{u\in\mathbb{Q}\cap[0,t]} B(u)$. Hence, S is an adapted process with non-decreasing paths. Let us recall the reflection principle for Brownian motion, see for instance Karatzas and Shreve (1988) p. 95.

Lemma 2 *Let $B = (B(t) : t \geq 0)$ denote standard Brownian motion ($B(0) = 0$ a.s.) and S its maximum process. For $y \geq 0$, $z > 0$*

$$P(B(t) < z - y, S(t) \geq z) = P(B(t) > y + z) \ . \tag{19}$$

Lemma 2 and equation (18) enable us to state the following reflection principle for symmetric α-stable Lévy motion.

Theorem 3 *Let Z_α be symmetric α-stable Lévy motion and $y \geq 0$, $z > 0$, then*

$$P\{Z_\alpha(t) < z - y, \sup_{0\leq s\leq t} Z_\alpha(s) \geq z\} \leq P\{Z_\alpha(t) > y + z\} \ . \tag{20}$$

If we take $y = 0$ in (20) and add to both sides $P\{Z_\alpha(t) > z\}$ we obtain the following corollary.

Corollary 1 *Let Z_α denote symmetric α-stable Lévy motion and let z be a positive number. Then*

$$P\{\sup_{0\leq s\leq t} Z_\alpha(s) \geq z\} \leq 2\,P\{Z_\alpha(t) > z\} \ . \tag{21}$$

Proof of Theorem 3. Consider

$$\begin{aligned}
P_t(z,y) &= P\{Z_\alpha(t) < z - y, \sup_{0\leq s\leq t} Z_\alpha(s) \geq z\} \\
&= P\{B(X(t)) < z - y, \sup_{0\leq s\leq t} B(X(s)) \geq z\} \\
&= \int_{D\times C} 1\{b(x(t)) < z - y, \sup_{0\leq s\leq t} b(x(s)) \geq z\}\ dM(x)\ dW(b) \\
&= \int_D dM(x) \int_C 1\{b(x(t)) < z - y, \sup_{0\leq s\leq t} b(x(s)) \geq z\}\ dW(b) \ ,
\end{aligned}$$

where the last equality follows from the independence of the two processes X and B. Here W denotes the Wiener measure on the space $C = C[0,\infty)$ of continuous functions and M is a measure on the space D induced by the process X. So

$$
\begin{aligned}
P_t(z,y) &= \int_D P\{B(x(t)) < z-y,\ \sup_{0\le s\le t} B(x(s)) \ge z\}\ dM(x) \\
&\le \int_D P\{B(x(t)) < z-y,\ \sup_{0\le s\le x(t)} B(s) \ge z\}\ dM(x)\ .
\end{aligned}
$$

Now using Lemma 2 we obtain

$$
\begin{aligned}
P_t(z,y) &\le \int_D P\{B(x(t)) > y+z\}\ dM(x) \\
&= \int_{D\times C} 1\{b(x(t)) > y+z\}\ dM(x)\ dW(b) \\
&= P\{B(X(t)) > y+z\}
\end{aligned}
$$

which ends the proof. □

We can now provide upper bounds for the ruin probability of the process Q, where the stable component Z_α is symmetric α-stable Lévy motion. If $X \sim S_\alpha(1,\beta,0)$ we denote by $G(x;\alpha,\beta)$ the cumulative distribution function of X. We set $\overline{G} = 1-G$.

Theorem 4 *Let Z_α be symmetric α-stable Lévy motion. For positive numbers u, c and λ we have*

$$
\begin{aligned}
P\{T(u+cs-\lambda^{1/\alpha}Z_\alpha(s)) \le t\} &\le 2\,P\{Z_\alpha(t) > u\lambda^{-1/\alpha}\} \\
&= 2\,\overline{G}(u/(\lambda t)^{1/\alpha};\alpha,0)\ .
\end{aligned}
$$

Proof. Using Corollary 1 we obtain

$$
\begin{aligned}
P\{T(u+cs-\lambda^{1/\alpha}Z_\alpha(s)) > t\} &= P\{\inf_{0\le s\le t}(u+cs-\lambda^{1/\alpha}Z_\alpha(s)) \ge 0\} \\
&= P\{\sup_{0\le s\le t}(-u-cs+\lambda^{1/\alpha}Z_\alpha(s)) \le 0\} \\
&= 1 - P\{\sup_{0\le s\le t}(-u-cs+\lambda^{1/\alpha}Z_\alpha(s)) > 0\} \\
&\ge 1 - P\{\sup_{0\le s\le t} Z_\alpha(s) \ge u\lambda^{-1/\alpha}\} \\
&\ge 1 - 2\,P\{Z_\alpha(t) > u\lambda^{-1/\alpha}\}\ .
\end{aligned}
$$

□

In order to prove an analogous result for the non-symmetric case, we need the following lemma:

Lemma 3 *Let Z_α be α-stable Lévy motion with $\alpha \ne 1$ and skewness parameter $|\beta| \le 1$. Then for $z > 0$ one has*

(i) $P\{Z_\alpha(t) > 0\} = \dfrac{1}{2} + \dfrac{1}{\pi\alpha}\arctan(\beta\tan\dfrac{\pi\alpha}{2}) =: \rho$,

(ii) $P\{\sup_{0\le s\le t} Z_\alpha(s) \ge z\} \le (\frac{1}{\rho})\, P\{Z_\alpha(t) > z\}$.

Proof. (i) Because of the self-similarity property we have $Z_\alpha(t) \overset{d}{=} t^{1/\alpha} Z_\alpha(1)$ and hence $P\{Z_\alpha(t) > 0\} = P\{Z_\alpha(1) > 0\} = \rho$ independent of t. From Zolotarev (1986) p. 79 we have

$$G(0;\alpha,\beta) = \frac{1}{2}\left(1 - \tilde\beta\,\frac{K(\alpha)}{\alpha}\right)$$

where $\tilde\beta$ is such that $\tan(\tilde\beta \frac{\pi K(\alpha)}{2}) = \beta \tan\frac{\pi\alpha}{2}$ and $K(\alpha) = \alpha - 1 + \mathrm{sign}(1-\alpha)$ (Samorodnitsky and Taqqu (1994) p. 8). It follows that

$$G(0;\alpha,\beta) = \frac{1}{2} - \frac{1}{\pi\alpha}\arctan(\beta\tan\frac{\pi\alpha}{2})$$

and therefore

$$\begin{aligned} P\{Z_\alpha(t) > 0\} = P\{Z_\alpha(1) > 0\} &= \overline{G}(0;\alpha,\beta) \\ &= \frac{1}{2} + \frac{1}{\pi\alpha}\arctan(\beta\tan\frac{\pi\alpha}{2}) \ . \end{aligned}$$

Note that in the case $\beta = 1,\ 1 < \alpha \le 2$ we have $G(0;\alpha,1) = 1/\alpha$.
(ii) Let τ be the first time the process Z_α crosses the level z, i.e. $\tau = \inf\{t > 0 : Z_\alpha(t) \ge z\}$. Then τ is a stopping time and $\{\sup_{0\le s\le t} Z_\alpha(s) \ge z\} = \{\tau \le t\}$. Write

$$\begin{aligned} &P\{\sup_{0\le s\le t} Z_\alpha(s) \ge z\} = \\ &\quad P\{\sup_{0\le s\le t} Z_\alpha(s) \ge z, Z_\alpha(t) > z\} + P\{\sup_{0\le s\le t} Z_\alpha(s) \ge z, Z_\alpha(t) \le z\} \end{aligned} \tag{22}$$

Because $\{Z_\alpha(t) > z\} \subseteq \{\sup_{0\le s\le t} Z_\alpha(s) \ge z\}$ the first summand on the right hand side of (22) reduces to $P\{Z_\alpha(t) > z\}$. Define $Z^*_\alpha(t) = Z_\alpha(\tau + t) - Z_\alpha(\tau)$. It is obvious that

$$\{\sup_{0\le s\le t} Z_\alpha(s) \ge z, Z_\alpha(t) \le z\} \subseteq \{\sup_{0\le s\le t} Z_\alpha(s) \ge z, Z^*_\alpha(t-\tau) \le 0\}$$

and hence, using the strong Markov property, we obtain

$$P\{\sup_{0\le s\le t} Z_\alpha(s) \ge z, Z_\alpha(t) \le z\} \le P\{\tau \le t\}(1-\rho) \ .$$

So

$$P\{\sup_{0\le s\le t} Z_\alpha(s) \ge z\} \le P\{Z_\alpha(t) > z\} + P\{\sup_{0\le s\le t} Z_\alpha(s) \ge z\}(1-\rho)$$

or equivalently

$$P\{\sup_{0\le s\le t} Z_\alpha(s) \ge z\} \le (\frac{1}{\rho})\, P\{Z_\alpha(t) > z\} \ .$$

$\square$

With the same arguments as in Theorem 4 we obtain an upper bound for the finite-time ruin probability in the non-symmetric case.

Theorem 5 *Let Z_α be α-stable Lévy motion with index $\alpha \neq 1$ and $|\beta| \leq 1$. For positive numbers u, c and λ we have*

$$\begin{aligned} P\{T(u + cs - \lambda^{1/\alpha} Z_\alpha(s)) \leq t\} &\leq (\frac{1}{\rho})\, P\{Z_\alpha(t) > u\lambda^{-1/\alpha}\} \\ &= (\frac{1}{\rho})\, \overline{G}(u/(\lambda t)^{1/\alpha}; \alpha, \beta) \ . \end{aligned}$$

Similar arguments as those used in the proof of Lemma 3 allow us to establish the following result.

Theorem 6 *Let Z_α be α-stable Lévy motion with index $\alpha \neq 1$ and $|\beta| \leq 1$ or $\alpha = 1$ and $\beta = 0$. For positive numbers u, c and λ we have*

$$P\{T(u + cs - \lambda^{1/\alpha} Z_\alpha(s)) \leq t\} \leq \frac{\overline{G}((u + ct)/(\lambda t)^{1/\alpha}; \alpha, \beta)}{\overline{G}(ct/(\lambda t)^{1/\alpha}; \alpha, \beta)} \ . \tag{23}$$

5 Numerical Results

In order to get some appreciation of the behaviour of the process Q defined in (4), we consider a classical risk process and its corresponding weak approximation. We first establish the theoretical result which permits us to compare the two processes. Finally, we consider a concrete example and use numerical methods to evaluate the finite-time ruin probabilities.

Let $(N(t) : t \geq 0)$ be a Poisson process with intensity λ. Define $Q^{(n)}$ and Q as in Section 2, namely

$$\begin{aligned} Q^{(n)}(t) &= u_0 + c^{(n)}t - \sum_{k=1}^{N(nt)} Y_k^{(n)} \ , \qquad \text{and} \\ Q(t) &= u_0 + \gamma t - \lambda^{1/\alpha} Z_\alpha(t) \ . \end{aligned}$$

Recall that condition (5) is fulfilled in the Poisson case. So, see Theorem 1, $Q^{(n)} \Rightarrow Q$ if and only if $\lim_{n\to\infty}(c^{(n)} - \lambda n \mu \varphi(n)) = \gamma$.
Define

$$\begin{aligned} \Psi^{(n)}(u_0, t_0) &= P\{T(Q^{(n)}) \leq t_0\} = P\{\inf_{0\leq s\leq t_0} Q^{(n)}(s) < 0\} \\ \Psi_W(u_0, t_0) &= P\{T(Q) \leq t_0\} = P\{\inf_{0\leq s\leq t_0} Q(s) < 0\} \ . \end{aligned}$$

Then, see Theorem 2, $\Psi^{(n)}(u_0,t_0) \to \Psi_W(u_0,t_0)$ as $n \to \infty$.

Consider now the classical risk process $R(t) = u + ct - \sum_{k=1}^{N(t)} Y_k$ with claim size distribution F and relative safety loading $\theta = \frac{c}{\lambda\mu} - 1 > 0$. Assume that F is in the domain of attraction of $S_\alpha(1,\beta,0)$. Denote the corresponding finite-time ruin probability by $\Psi(u,t)$. Then we have, for each $n \in \mathbb{N}$,

$$\begin{aligned}
\Psi(u,t) &= P\left\{\inf_{0\le s\le t}\left(u + cs - \sum_{k=1}^{N(s)} Y_k\right) < 0\right\} \\
&= P\left\{\inf_{0\le s\le t}\left(\frac{u}{\varphi(n)} + \frac{cs}{\varphi(n)} - \frac{1}{\varphi(n)}\sum_{k=1}^{N(s)} Y_k\right) < 0\right\} \\
&= P\left\{\inf_{0\le s\le \frac{t}{n}}\left(\frac{u}{\varphi(n)} + \frac{cns}{\varphi(n)} - \frac{1}{\varphi(n)}\sum_{k=1}^{N(ns)} Y_k\right) < 0\right\} .
\end{aligned}$$

Put $\gamma = \theta\lambda n\mu/\varphi(n)$, $u_0 = u/\varphi(n)$ and $t_0 = t/n$. This leads to the weak approximation

$$\Psi(u,t) \approx \Psi_W(u_0,t_0) . \tag{24}$$

We say that F is in the normal domain of attraction of the stable law if $\varphi(n) = n^{1/\alpha}$. In this case it follows that

$$\Psi_W(u_0,t_0) = P\{\inf_{0\le s\le t}(u + \lambda\mu\theta s - \lambda^{1/\alpha} Z_\alpha(s)) < 0\} . \tag{25}$$

In Figures 3 and 4 the simulated sample paths of the processes R and Q are compared. An explicit formula for $\Psi(u,t)$ can usually only be found in terms of Laplace transforms. Thorin and Wikstad (1973) proceed in this way and evaluate $\Psi(u,t)$ numerically by inverting a double Laplace transform. In our approach we use the simple Monte Carlo method to estimate both $\Psi(u,t)$ as well as the finite-time ruin probability of our weak approximation. For simulations of stable variables and processes we refer to Weron (1996). Where comparisons are possible, our simulation results agree with the 'exact' values of Thorin and Wikstad (1973). As an example, we consider Pareto distributed claims, i.e.

$$F(x) = 1 - \left(\frac{\nu}{\nu + x}\right)^\alpha \qquad x \ge 0, \quad 1 < \alpha < 2.$$

The mean μ is given by $\nu/(\alpha-1)$. Because $x^\alpha(1-F(x))$ tends to the constant value ν^α as $x \to \infty$, we conclude that F is in the normal domain of attraction of $S_\alpha(1,1,0)$ (see Feller (1971) p. 581). In our simulations ν equals $1/2$ and $\alpha \in \{1.2, 1.5, 1.8\}$, the latter values are typical for heavy–tailed insurance portfolios, see Embrechts et al. (1996). The relative safety loading θ is assumed to be 5%. We ran 100000 simulations in order to estimate $\Psi(u,t)$ and $\Psi_W(u,t)$ for $t = 1000$ and $t = 10000$,

Figure 3: Simulations of a classical risk process with unit intensity and Pareto distributed claims ($\nu = 0.5$, $\alpha = 1.5$), initial capital $u = 100$ and safety loading $\theta = 0.05$.

Figure 4: Simulations of the corresponding weak approximation Q .

respectively. The relative error ε is given by $\varepsilon = (\hat{\Psi}_W - \hat{\Psi})/\hat{\Psi}$.

Table 1

Table 2

In Tables 1 and 2 we have presented some numerical values for illustrative purposes. For the chosen parameters the approximation mostly overestimates the 'exact' ruin probabilities. Small values of α lead to smaller values of the relative error ϵ . This can be explained by the fact that convergence in (5) is faster for small values of α (see also (13)). Further numerical work has shown that the upper bounds derived in Theorems 5 and 6 are very crude.

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