

GEOMETRIC AND ANALYTIC GROUP THEORY
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STATEMENT. I am currently interested in bounded cohomology and uniformly finite homology.

For an introduction to *uniformly finite homology*, see Francesca Diana's research statement. Recently, Francesca and I have shown that the uniformly finite homology groups of amenable groups are infinite dimensional in many cases.

In order to construct *bounded cohomology*, instead of looking at general cochains as in regular cohomology of topological spaces, one only considers bounded cochains (with respect to the canonical ℓ^1 -norm on the chain complex). This leads to a theory very different from ordinary cohomology.

First, one notes that bounded cohomology encompasses the so called simplicial volume of a closed oriented manifold, which in important cases corresponds to the Riemannian volume of the manifold. On the other hand, bounded cohomology essentially depends only on the fundamental group of the corresponding space and can thus be viewed as a purely group theoretical tool. Hence bounded cohomology provides a link between (Riemannian) geometry, topology and group theory. This is exemplified in the crucial role it plays in Gromov's proof of Mostow's Rigidity Theorem.

In the calculations of bounded cohomology, ideas from geometric group theory are often central. Conversely, bounded cohomology can for instance detect whether a group is hyperbolic (or amenable) or not.

The theory is both enriched and complicated by the lack of accessible tools (like excision) for more general calculations. A fundamental exception to this is the existence of a relative version of bounded cohomology, essentially due to Gromov. Though there exists now a variety of constructions of relative bounded cohomology, there seems to be no optimal one.

We have designed a new approach to relative bounded cohomology that should be at least complementary to the existing ones. We are now establishing a homological algebraic description of resolutions that should calculate relative bounded cohomology and the correct norm.

The script by C. Löh is a great starting point for getting acquainted with the subject. It also contains a comprehensive bibliography. The pivotal point of the subject remains the fundamental work of M. Gromov.

REFERENCES

- [1] M. Blank, F. Diana, *Uniformly finite homology of amenable groups*, Preprint arXiv (09.2013).
- [2] M. Gromov, *Volume and bounded Cohomology*, Publ. Math. IHES, 56 (1982).
- [3] C. Löh, *Group Cohomology and Bounded Cohomology. An Introduction for Topologists*, Lecture Notes (2010).
Available online via www.mathematik.uni-regensburg.de/loeh/teaching/topologie3_ws0910/prelim.pdf.

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