RESEARCH STATEMENT

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In my MSc thesis I studied the irreducibility of some representations of subgroups of the group $\mathrm{Diff}_c(M)$ of compactly supported diffeomorphisms of a smooth manifold M. If we take a measure μ on M, having smooth density with respect to the Lebesgue measure in maps, we may define a unitary representation π of $\mathrm{Diff}_c(M)$ on $L^2(M,\mu)$ by

$$\pi_s(\phi)f = \left(\frac{d\phi_*\mu}{d\mu}\right)^{1/2+is} f \circ \phi^{-1}, \tag{1}$$

where $s \in \mathbb{R}$.

The case of the group of compactly supported diffeomorphisms preserving a measure was described by Vershik, Gelfand and Graev. In my thesis I considered the irreducibility of representations (1) for the groups $\mathrm{Sympl}_c(M)$ and $\mathrm{Cont}_c(M)$ of compactly supported symplectomorphisms and contactomorphisms.

After the "large" groups of diffeomorphisms, I studied representations of Thompson's groups F and T, which are finitely presented, hence "small" in the combinatorial sense. Their natural actions on the unit interval and unit circle however still resemble the action of a "large" group, and the representations (1) of F and T turn out to be irreducible. Moreover, representations π_s and π_t are inequivalent, provided that $s-t$ is not a multiple of $2\pi/\log 2$.

In the representation theory of $SL_2(\mathbb{R})$, the representations of the form (1), associated to the natural action of $SL_2(\mathbb{R})$ on $\mathbb{P}(\mathbb{R}^2)$, form a part of the principal series. They are induced from one-dimensional representations of subgroups of $SL_2(\mathbb{R})$. This is not the case for the Thompson's groups. In fact, π_s are nonequivalent to representations induced from finite-dimensional representations of proper subgroups of F or T. Hence, the two possible generalizations of the principal series to F and T are disjoint.

One of the problems I am working on is to generalize the above results for quasi-invariant actions of discrete groups on arbitrary measure spaces. In other words, I want to understand how the irreducibility of the representations π_s is related to dynamical properties of the action.

My work on representations of the group of contactomorphisms has inspired the following question:

Problem. *Let G be a topological group. Suppose that G contains no nontrivial compact subgroups. Does it imply that the convolution algebra $\mathcal{M}_c(G)$ of compactly supported complex Borel measures on G has no zero divisors?*

The positive answer in the case of $\mathbb{R}^n$ is a variant of the Titchmarsh convolution theorem. The answer for locally compact abelian groups follows from the work of Benjamin Weiss. In a recent paper I managed to give a positive answer for supersolvable Lie groups. I want to continue this work and extend my results to a wider class of groups.

All my papers and preprints can be found in arXiv.