Research description

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I'm working in the field of Metric Geometry, my main interest is the Nagata dimension.

Definition 1. Let X be a metric space. Then, the *Nagata dimension* of X (denoted by $\dim_N X$) is the least integer n, such that for all $R > 0$ there is a CR-bounded cover of X with R-multiplicity at most $n + 1$ and the number $C > 0$ is independent of R.

There are also small-scale ($\ell-\dim X$) and large-scale ($\ell-\text{asdim}\, X$) versions of the Nagata dimension. We have that $\dim_N X = \sup\{\ell-\dim X, \ell-\text{asdim}\, X\}$. Other useful properties include ($\dim X$ denotes the topological dimension of X):

- For $A \subset \mathbb{R}^n$ with nonempty interior, we have $\dim_N A = n$;
- $\dim_N (X \times Y) \leq \dim_N X + \dim_N Y$;
- For $X = Y \cup Z$, we have $\dim_N X = \sup\{\dim_N Y, \dim_N Z\}$;
- $\dim X \leq \dim_N X$;
- If X, Y are metric spaces an $f : X \to Y$ is a quasisymmetric homeomorphism, then $\dim_N X = \dim_N Y$.

There are some interesting connections between the Nagata dimension and Lipschitz maps, see [1].

References

[1] U. Lang and T. Schlichenmaier, *Nagata Dimension, Quasisymmetric Embeddings, and Lipschitz Extensions.* IMRN 2005, no 58, 3625-3655.