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Here is a brief description of our recent joint work with Daniel T. Wise on separability of embedded surfaces in 3–manifolds.

A subgroup $H \subset G$ is *separable* if H equals the intersection of finite index subgroups of G containing H. Scott proved that if $G = \pi_1 M$ for a manifold M with universal cover $\widetilde{M}$, then H is separable if and only if each compact subset of $H\backslash\widetilde{M}$ embeds in an intermediate finite cover of M [Sco78, Lem 1.4]. Thus, if $H = \pi_1 S$ for a compact surface $S \subset H\backslash\widetilde{M}$, then separability of H implies that S embeds in a finite cover of M. Rubinstein–Wang found a properly immersed π_1–injective surface $S \looparrowright M$ in a graph manifold such that S does not lift to an embedding in a finite cover of M, and they deduced that $\pi_1 S \subset \pi_1 M$ is not separable [RW98, Ex 2.6].

Our main result is:

Theorem 1. *Let M be a compact connected 3–manifold and let $S \subset M$ be a properly embedded connected π_1–injective surface. Then $\pi_1 S$ is separable in $\pi_1 M$.*

The problem of separability of an embedded surface subgroup was raised for instance by Silver–Williams — see [SW09] and the references therein to their earlier works. The Silver–Williams conjecture was resolved recently by Friedl–Vidussi in [FV12], who proved that $\pi_1 S$ can be separated from some element in $[\pi_1 M, \pi_1 M] - \pi_1 S$ whenever $\pi_1 S$ is not a fiber.

We proved Theorem 1 when M is a graph manifold in [PW11, Thm 1.1]. Theorem 1 was also proven when M is hyperbolic [Wis11]. In fact, every finitely generated subgroup of $\pi_1 M$ is separable for hyperbolic M, by [Wis11] in the case $\partial M \neq \emptyset$ and by Agol's theorem [Ago12] for M closed.

References

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