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I am interested in quasimorphisms and quasicocycles. The following is an idea for constructing a quasimorphism starting from a free group automorphism: Let $\mathbb{F}_n$ be the free group of rank $n \geq 3$, and let $\varphi \in \mathrm{Aut}(\mathbb{F}_n)$ such that

(i) The mapping torus $\Gamma_\varphi := \mathbb{F}_n \rtimes_\varphi \mathbb{Z}$ is word-hyperbolic, and

(ii) The abelianization $\varphi^{\mathrm{ab}} \in \mathrm{GL}(n, \mathbb{Z})$ fixes a non-zero vector in $\mathbb{Z}^n$.

Such automorphisms exist by the work of Clay-Pettet (see [1]). Using the Mayer-Vietoris sequence for the cohomology of HNN extensions one can see that condition (ii) is equivalent to the existence of a non-zero class $\omega \in \mathrm{H}^2(\Gamma_\varphi, \mathbb{Z})$. As Γ_φ is hyperbolic, a theorem of Neumann-Reeves (see [3]) says that $\omega = [c]$ for some bounded 2-cocycle c, so that $[c]$ can be seen as a non-vanishing class in the bounded cohomology $\mathrm{H}^2_{\mathrm{b}}(\Gamma_\varphi, \mathbb{Z})$. A theorem of Gromov (see [2]) implies that the restriction map $\mathrm{H}^2_{\mathrm{b}}(\Gamma_\varphi, \mathbb{Z}) \longrightarrow \mathrm{H}^2_{\mathrm{b}}(\mathbb{F}_n, \mathbb{Z})$ is injective, which means that $[c|_{\mathbb{F}_n}]$ is a non-trivial class in $\mathrm{H}^2_{\mathrm{b}}(\mathbb{F}_n, \mathbb{Z})$. Since $\mathrm{H}^2(\mathbb{F}_n, \mathbb{Z}) = 0$ we have $c|_{\mathbb{F}_n} = \partial f$ for some non-trivial quasimorphism $f : \mathbb{F}_n \longrightarrow \mathbb{Z}$. I would like to understand the properties of such an f. What does f know about the automorphism φ ? Can one relate the defect of f, or the Gromov norm of its class, to some quantity associated to φ ?

References:

[1] M. Clay and A. Pettet. Current twisting and nonsingular matrices. *Comment. Math. Helv.*, 87(2):385–407, 2012.

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[3] W. D. Neumann and L. Reeves. Central extensions of word hyperbolic groups. *Ann. of Math. (2)*, 145(1):183–192, 1997.