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Research description

My current research interests are combinatorial and geometric group theory. The first topic is the study of growth functions of the groups. While several remarkable results by Gromov, Grigorchuk and J.Wilson gave beautiful insights about the hierarchy of the growth functions, there are still many interesting problems to be solved. My former results were related to the study of asymptotic behaviour of $f_{G,S}(n)$ and its dependence on the set S. The limit $\lambda(G,S) = \lim_{n\to\infty} \sqrt[n]{f_{G,S}(n)}$ is called an exponential growth rate and it roughly describes the behaviour of $f_{G,S}(n)$. Even for some "easy" groups it can be hard to compute $\lambda(G,S)$, for example finding this constant for the Baumslag-Solitar group $BS(2,3) = \langle a, t \mid ta^2t^{-1} = a^3 \rangle$ is a challenge. Even when it is possible to compute the value $\lambda(G,S)$ for some generating set S, it can be quite complicated to prove that this value is equal to $\lambda(G) = \inf\{\lambda(G,S')\}$, where S' runs along all finite sets that generate G. My former works were about lower and upper bounds for $\lambda(G)$ for some classical groups like some one-relator groups and some free products. One of interesting question here is the value of $\lambda(\pi_1(\Sigma_g))$ for the the fundamental group of the closed orientable surface Σ_g of genus $g \geq 2$. Another exciting problem related to growth which attracts me a lot is the question whether exponentiality of the growth necessarily forces the Cayley graph of the group to have isometrically embedded 3-trees (for non-amenable groups it was proved by Benjamini and Schramm, for the solvable groups – by de Cornulier and Tessera).

In the recent work with Tobias Hartnick we studied quasimorphisms of the free group F_n (also called quasicharacters). The linear space of quasimorhisms of a group is often almost the same as a space $H^{(2)}_{b,2}(G)$ of the bounded cohomologies of G. In 80's Brooks introduced nice infinite series of quasimorphisms for F_n which he called counting quasimorphisms. Brooks also claimed a linear independency of counting quasimorphisms, this was shown to be false as R.I.Grigorchuk gave an explicit relation in his the notable paper "Some results on bounded cohomology" (1994). But even though there is a linear dependency, the span $\mathfrak{Q}(F_n)$ of all counting quasimorphisms is infinite-dimensional and this space was the subject of our study. In fact this space has a very natural description as a graded sequence of linear finite-dimensional spaces. We managed to find a nice complete set of relations for the space $\mathfrak{Q}(F_n)$. In future we plan to use this presentation to study the action of $\mathrm{Aut}(F_n)$ on $\mathfrak{Q}(F_n)$, previously discussed by Hartnick and Schweitzer.

Besides the discussed questions I am quite interested in decision problems related to group and semigroup theory and also in the questions of computational complexity for the problems that arise in this field. The question that I aim at here is the complexity of quadratic equations for finite groups.