Research Description

Anne Thomas, University of Glasgow

August 12, 2013

My research lies at the junction of geometric group theory and the topics of lattices, rigidity and buildings. There is a long history of studying lattices in Lie groups, but little is known about lattices in more general locally compact groups.

Let G be a locally compact group. A discrete subgroup $\Gamma \leq G$ is a *lattice* if $\Gamma\backslash G$ carries a finite G-invariant measure, and is *cocompact* or *uniform* if $\Gamma\backslash G$ is compact. The key cases are:

1. Lie groups, such as $G = \mathrm{SL}_n(\mathbb{R})$. Lattices in G are often studied by considering the associated symmetric space.

2. Algebraic groups over nonarchimedean local fields, such as $G = \mathrm{SL}_n(\mathbb{Q}_p)$. Lattices in G are often studied by considering the associated Bruhat–Tits building X, which is a locally finite polyhedral complex. The group G is closed and either finite index or cocompact in the automorphism group $\mathrm{Aut}(X)$.

3. Complete Kac–Moody groups over finite fields, the first example of which is $G = \mathrm{SL}_n(\mathbb{F}_q((t)))$ (most such G are known to be nonlinear; they may be thought of as "infinite-dimensional Lie groups"). Lattices in G are often studied by considering the associated Tits building X, which is a locally finite polyhedral complex. The group G is closed in $\mathrm{Aut}(X)$, but not in general cocompact.

4. Let X be a locally finite polyhedral complex. There are many fascinating examples, including trees and other graphs, products of trees, buildings or, for example, polygonal complexes with all vertex links the Petersen graph. The full automorphism group $G = \mathrm{Aut}(X)$ is naturally a locally compact group.

I mainly work on cases 3 and 4. Many questions from the "classical" cases 1 and 2 make sense in these contexts, and may be re-understood from a more geometric point of view. New phenomena also occur, as seen in the rich theory of tree lattices and when X is a product of trees.

In the course of my research on lattices, I have investigated the Davis complex associated to a Coxeter system, and learnt how these Davis complexes, which are the apartments, may be glued together to form a building. I am now using this understanding to study the quasi-isometric classification of right-angled Coxeter groups, and questions from algebraic combinatorics concerning infinite non-affine Coxeter groups and the group $G = \mathrm{SL}_n(\mathbb{F}_q((t)))$.